DOCUMENT RESUME

ED 085 548 CE 000 728

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TITLE	Geometry: Career Related Units. Teacher's Edition.
INSTITUTION	Minnesota State Dept. of Education, St. Paul. Div. of Vocational and Technical Education.; Robbinsdale Independent School District 281, Minn.
PUB DATE	Jan 73
NOTE	270p.
EDRS PRICE	MF-$0.65 HC-$9.87
DESCRIPTORS	*Career Education; *Curriculum Enrichment; *Geometry; *High School Students; Resource Units; Senior High Schools; *Unit Plan

ABSTRACT

Using six geometry units as resource units, the document explores 22 math-related careers. The authors intend the document to provide senior high school students with career orientation and exploration experiences while they learn geometry skills. The units are to be considered as a part of a geometry course, not a course by themselves. The six geometry units (right triangles and the Pythogorean theorem, polygons and their areas, parallel lines, standard constructions, volume, and circle relationships) may either be studied first or used as resource units as the student works in any of the career units: printing and the graphic arts, heavy equipment operator, fashion and apparel design, navigation, painting and paperhanging, landscape technology, carpenter, architecture and drafting, optical technician, sheet metal, engineering, machinist, cement workers, forestry, electrician, general contractor, home planning, cabinetmaking, plumbing and pipe fitting, surveyor, outdoor advertising, and space. The teacher's edition contains an answer key. (AG)

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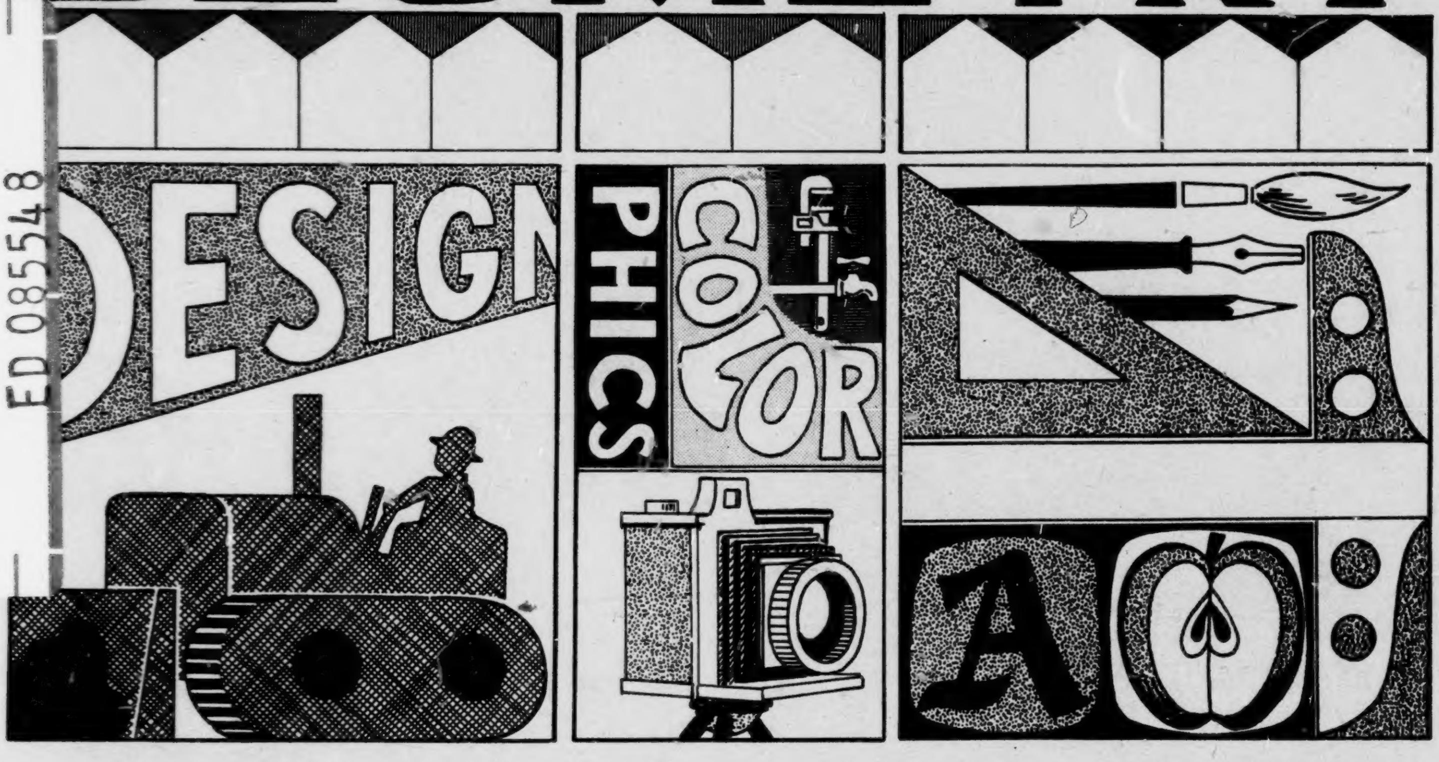
GEOMETRY



CAREER RELATED UNITS

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MINNESOTA STATE DEPARTMENT OF EDUCATION

VOCATIONAL-TECHNICAL DIVISION

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CAPITOL SQUARE BUILDING
SAINT PAUL, MINNESOTA

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DEAR EDUCATOR:

THIS DOCUMENT IS PART OF A PROJECT FUNDED BY THE MINNESOTA DEPARTMENT OF EDUCATION, DIVISION OF VOCATIONAL-TECHNICAL EDUCATION. IT WAS DEVELOPED TO SHOW HOW GEOMETRY CURRICULUM CAN BE ENHANCED BY ADDING REAL-LIFE CAREER ORIENTED AREAS REQUIRING MATH SKILLS.

IT SHOULD PROVIDE STUDENTS WITH CAREER ORIENTATION AND EXPLORATION EXPERIENCES AS WELL AS BEING AN EFFICIENT, RELEVANT TEACHING VEHICLE FOR HELPING STUDENTS ENJOY LEARNING GEOMETRY SKILLS.

RMMadson

Robert M. Madson, Director
Program Operations
DIVISION OF VOCATIONAL-TECHNICAL EDUCATION

RMM:LBK/sns

I. GEOMETRY RESOURCE UNITS -- TABLE OF CONTENTS

1. Right Triangles and the Pythagorean Theorem
2. Polygons and Their Areas
3. Parallel Lines
4. Standard Constructions
5. Volume
6. Circle Relationships

GEOMETRY -- CAREER RELATED UNITS

Teacher's Edition

A Joint Project between the Minnesota State Department of
Vocational Education and the Robbinsdale Area Schools

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INTRODUCTION

This project is an attempt to bring a practical approach to a senior high school geometry course. At the onset of the project, this seemed like a simple matter. The writing team began their work by investigating many careers. They found many isolated cases where geometry is used, but not in sufficient quantity to build a unit. This forced the team, in some cases, to work with broader categories of careers.

The units produced were written using as practical an approach as possible, and only after interviewing a person actually involved in that job. The purpose of the interview was to find the geometry-related tasks of each career.

The total package turned out to be a set of consumable units written in narrative form including six geometry resource units and 22 career units. The units are intended to be used by the students, written in, and kept. Note: These units are meant to be used as a part of a geometry course and are not meant to be used exclusively by themselves.

PROCEDURE

There are several procedures that could be followed in using these materials. One would be to work through all of the resource units with the class and then have students as individuals select the career units they are interested in. A second way would be to allow the students as individuals to begin work in the career units and then refer to the resource units as they are needed.

Evaluation: A most successful way of dealing with units of this type is to assign each unit a point value. A student then must earn a certain minimum number of points working through units of his choice.

CAREER-RELATED GEOMETRY

Answer Keys: The answer keys to these units should be made available to the student by the units, not in book form. This keeps the student from checking the answers for one unit and copying the answers to his next unit.

Resource Materials: Resource materials of value would be:

Occupational Outlook Handbook

Encyclopedia of Careers, Volumes 1 and 2

LIST OF UNITS

I. Geometry resource units

1. Right Triangles and the Pythagorean Theorem
2. Polygons and Their Areas
3. Parallel Lines
4. Standard Constructions
5. Volume
6. Circle Relationships

II. Career units

1. Printing and Graphic Arts
2. Heavy Equipment Operator
3. Fashion and Apparel Design
4. Navigation
5. Painting and Paperhanging
6. Landscape Technology
7. Carpenter
8. Architecture and Drafting
9. Optical Technician
10. Sheet Metal
11. Engineering
12. Machinist
13. Cement Worker
14. Forestry
15. Electrician
16. General Contractor
17. Home Planning
18. Cabinetmaking
19. Plumbing and Pipe Fitting
20. Surveyor
21. Outdoor Advertising
22. Space

STUDENT RECORD SHEET

I. Geometry Resource Units

____
Name

Unit	Date Started	Date Finished	Additional Math Work	Related Projects	Comments
1. Right Triangles and the Pythagorean Theorem					
2. Polygons and Their Areas					
3. Parallel Lines					
4. Standard Constructions					
5. Volume					
6. Circle Relationships					

STUDENT RECORD SHEET

II. Career Units

____
Name

Unit	Date Started	Date Finished	Additional Math Work	Related Projects	Comments
1. Printing					
2. Heavy Equipment Operator					
3. Fashion and Apparel Design					
4. Navigator					
5. Painting and Wall Papering					
6. Landscape Technology					
7. Carpenter					
8. Architecture and Drafting					
9. Optical Technician					
10. Sheet Metal					
11. Engineering					
12. Machinist					
13. Cement Worker					
14. Forestry					
15. Electrician					
General Contractor					

STUDENT RECORD SHEET

II. Career Units

____
Name

Unit	Date Started	Date Finished	Additional Math Work	Related Projects	Comments
17. Home Planning					
18. Cabinetmaking					
19. Plumbing and Pipe Fitting					
20. Surveyor					
21. Outdoor Advertising					
22. Space					

APPENDIX:
ANSWER KEYS

RIGHT TRIANGLES AND THE PYTHAGOREAN THEOREM--ANSWER K

1. a. 90°

   b. Right triangle

   c. Yes $9^2 + 12^2 = 15^2$
   $81 + 144 = 225$

2. a. $\angle C$

   b. AB or C

   c. Right; Hypotenuse

3. a. No

   b. Yes

4. a. 20

   b. 10

   c. $10 \cdot \sqrt{3}$

5. a. $c = 7 \cdot \sqrt{2}$

   b. $x = \frac{25}{\sqrt{2}}$ or $\frac{25\sqrt{2}}{2}$

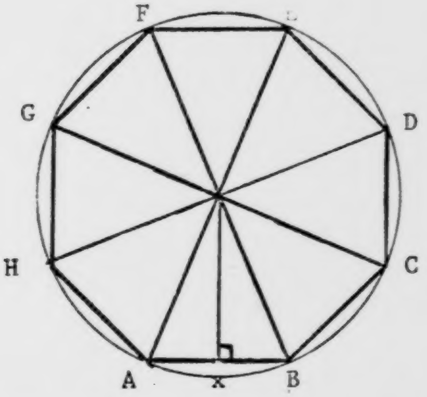
POLYGONS AND THEIR AREAS -- ANSWER KEY

Type of Polygon	Sites	△'s Formed	Degrees in Each △	Total Degrees in the Polygon
triangle	3	1	180°	180°
quadrilateral	4	2	180°	360°
pentagon	5	3	180°	540°
hexagon	6	4	180°	720°
octagon	8	6	180°	1080°
decagon	10	8	180°	1440°
n-gon	N	N-2	180°	(N-2)•(180)

1. 900° square

2. a. 108°
   b. 135°

3. 45°



4. Area = 15.2 square inches   5. Area = 52.5 square inches

6. 369.5 square inches

7. 32 square inches

8. 320 square inches

PARALLEL LINES -- ANSWER KEY

1. a. 50°

   b. 130°

   c. 50°

   d. 130°

   e. 50°

   f. 130°

   g. 50°

   h. 130°

2. a. $\angle 6$ or $\angle 4$

   b. $\angle 4$ or $\angle 6$

   c. Equal

3. a; b. $\angle 2$ and $\angle 6$

   c; d. $\angle 3$ and $\angle 7$

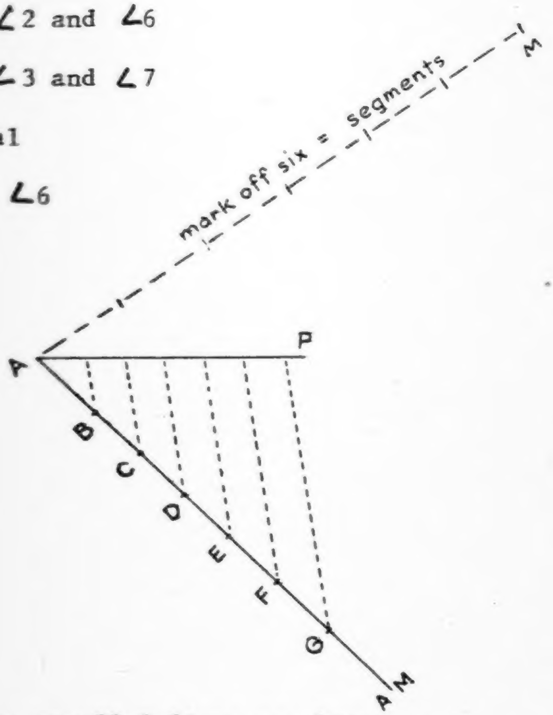
   e. Equal

4. $\angle 3$ and $\angle 6$

5. Yes

6. 6

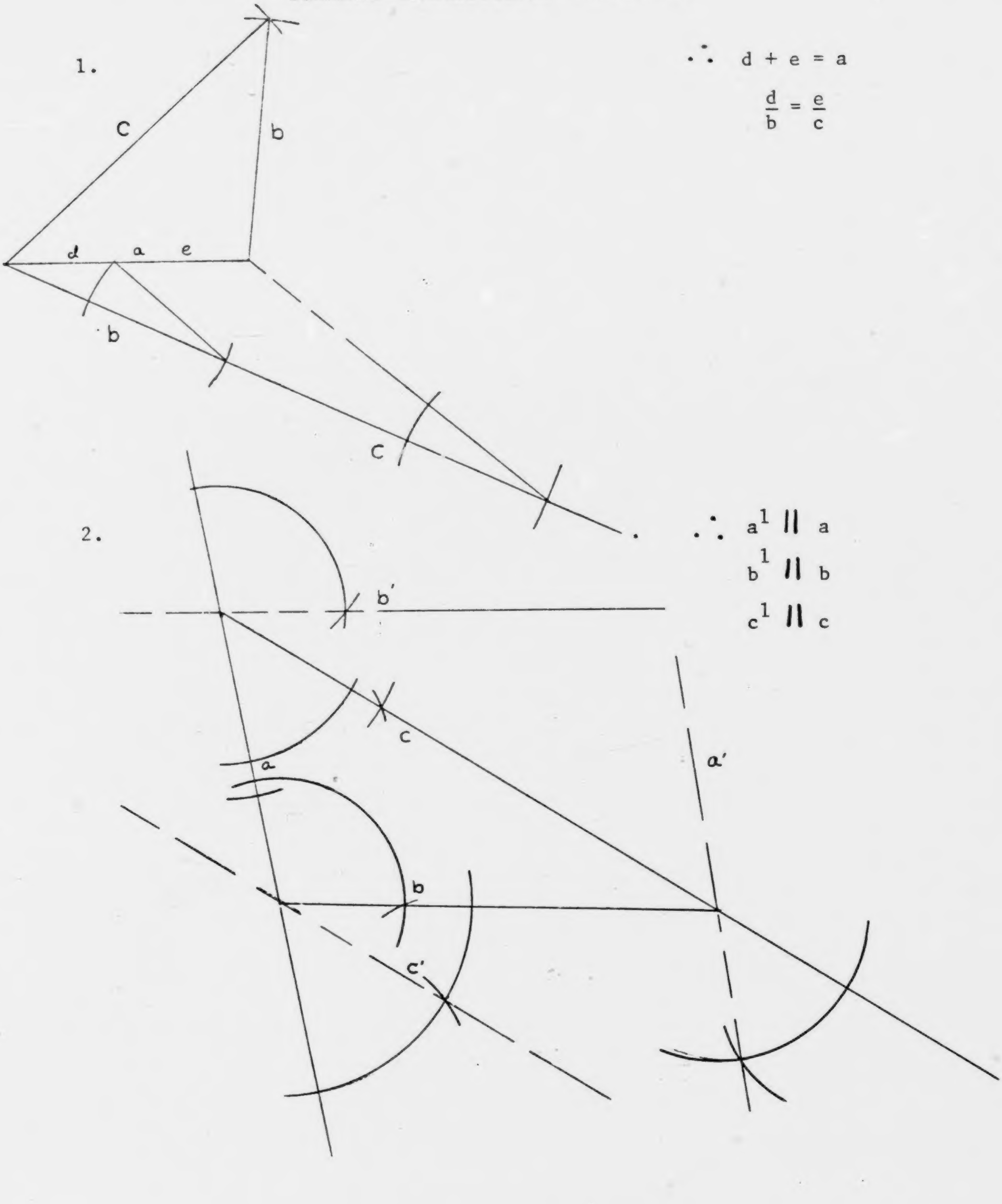
7.



Construct parallel lines as shown.
To divide AB draw another line similar to AG and join its endpoint M with B.
Then construct // lines.

STANDARD CONSTRUCTIONS--ANSWER KEY

1.



$\therefore\ d + e = a$

$\frac{d}{b} = \frac{e}{c}$

2.

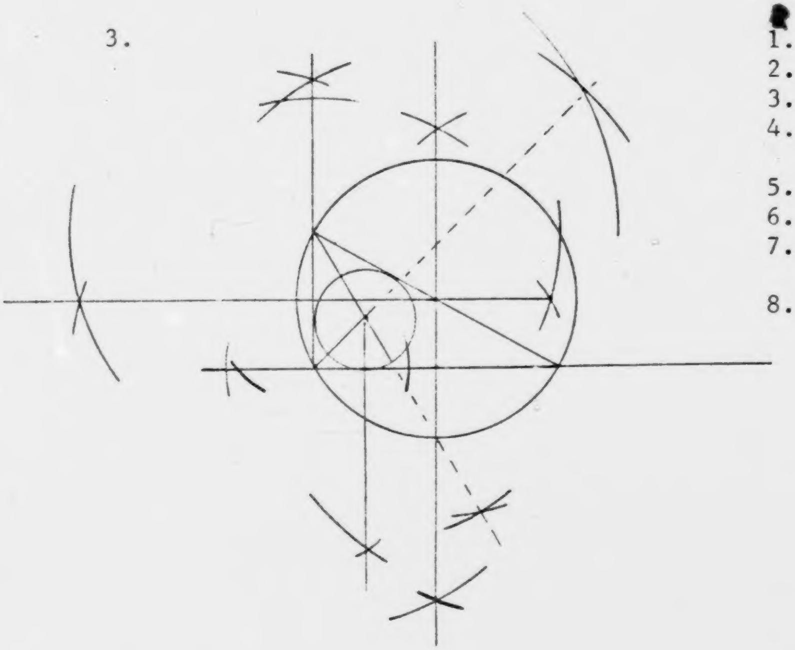
$\therefore\ a^1 \parallel a$

$b^1 \parallel b$

$c^1 \parallel c$

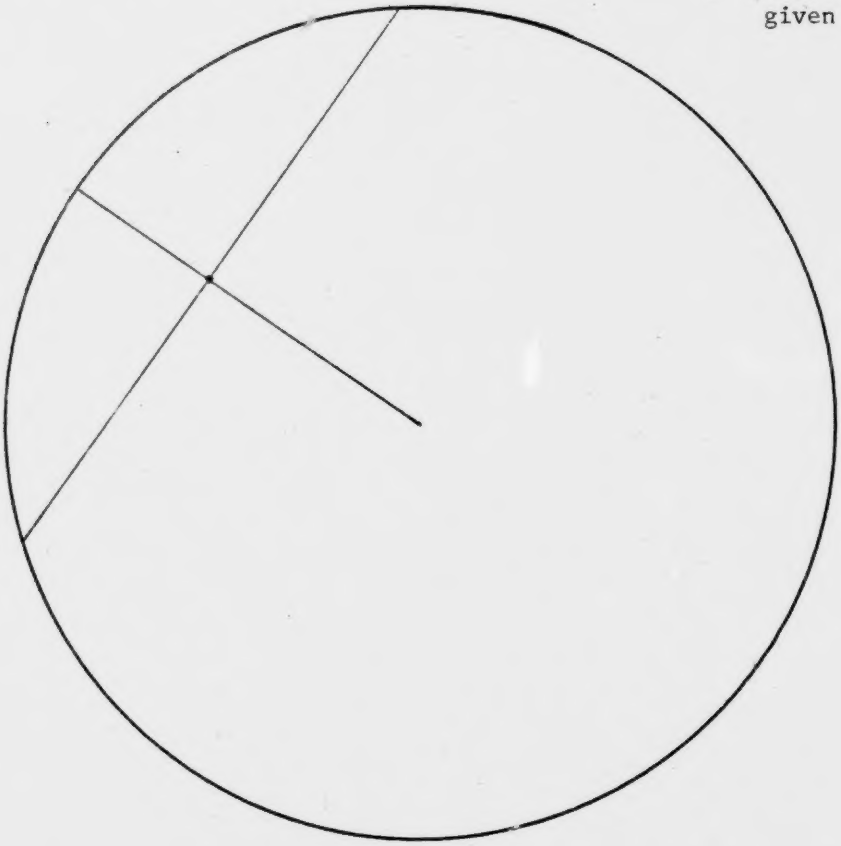
STANDARD CONSTRUCTIONS--ANSWER KEY

3.



1. Construct ⊥ angle.
2. Construct hypotenuse = 2 x short leg
3. ∴ 30°-60°-90°
4. Construct 2 ∠-bisectors for inscribed center
5. Drop ⊥ for radius length.
6. Draw semicircle
7. Construct 2 ⊥- biscetors of sides for circumscribed center
8. Draw circumscribed ⊙

4.



1. Construct radius through given point.
2. Construct to radius at the given point.

VOLUME -- ANSWER KEY

IV. a. 106.6 cubic yards

b. 5000 cubic feet

c. 185.2 cubic yards

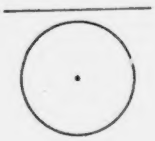
V. $V = B \times h$

$= \pi \cdot 6^2 \cdot 18$

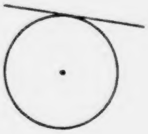
$= \frac{22}{7} \times \frac{36}{1} \times \frac{18}{1} = 2036.57$ cubic inches

CIRCLE RELATIONSHIPS -- ANSWER KEY

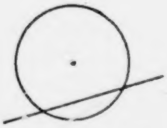
1. a.



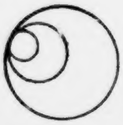
b.



c.



2. a.



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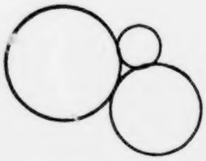
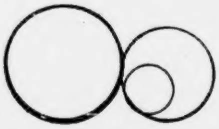
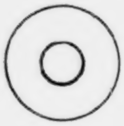


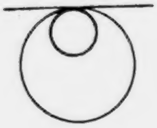
c.



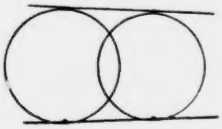
3. a.



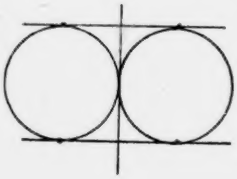
b.



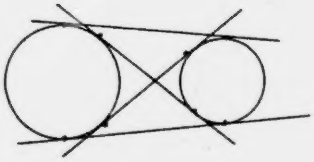
c.



d.



e.



4. a. 45
   b. 90
   c. 45
   d. 30

5. a. 80
   b. 50

6. 37

7. 125

8. a. 6.98
   b. 18.32
   c. 12.56
   d. 1,308.33

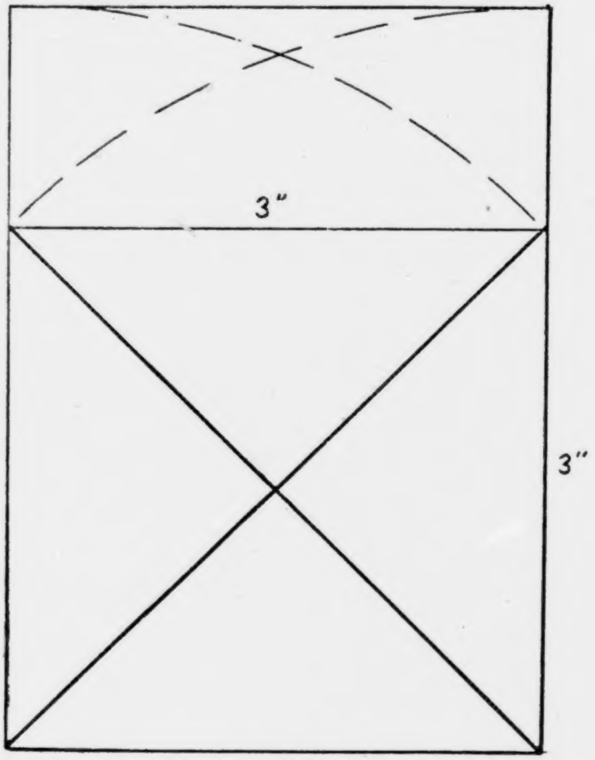
9. a. 33.49
   b. 83.73
   c. 50.24
   d. 89.32

10. a. 41.04
    b. 1.45
    c. 8.26
    d. 2.46

11. 10.08

PRINTING AND GRAPHIC ARTS -- ANSWER KEY

1. Root -2, hypotenuse
2. Root-4, long
3. Root-3, printers
4. Golden
5. Root-2, hypotenuse
6. 9" x 6" -- printers

   $2\frac{1}{2}$ x $3\frac{1}{2}$ -- hypotenuse
7. (As on page 4)
8. 

HEAVY EQUIPMENT OPERATOR -- ANSWER KEY

1. 56' x 48' = 2688 square feet

2. 21,504 cubic feet = 796.4 cubic yards

3. 208 feet

4. V = 208' x 1' x $\frac{10'}{12}$ = 173.3 cubic feet

5. 21,677.3 cubic feet or 802.86 cubic yards

6. A = 54' x 11' = 594 square feet

7. A = 44' x 18' = 792 square feet = IJKL

8. Area of circle = 1,017.4

   Area of sector = 254.3 square feet

9. $792 - 508.6 = 283.4 \doteq 283$ square feet

10. Total area of driveway = 2,971 square feet

11. 1485.5 cubic feet (V = 2,971 x .5)

12. 55 cubic feet

13. Hours = 5.5

    Cost = $247.50

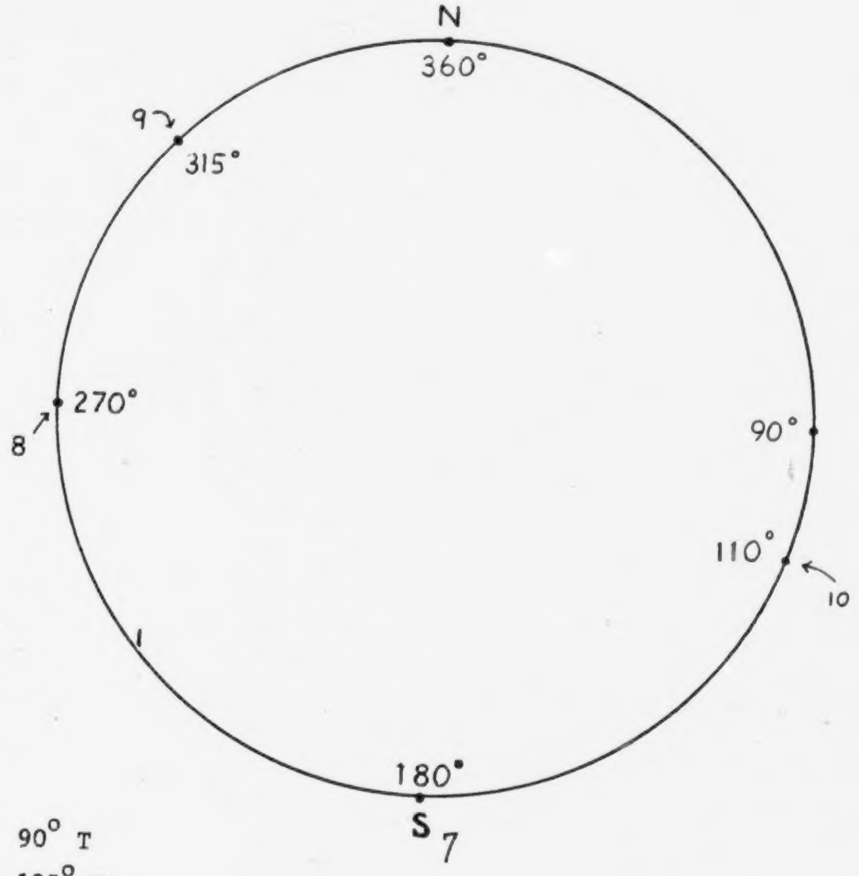
14. 495 cubic feet or 18.33 cubic yards

FASHION AND APPAREL DESIGN -- ANSWER KEY

1. 16 inches
2. 9 inches
3. Drawings vary.
4. Examples vary.
5. Examples vary.
6. 1.618
7. Examples vary.
8. Examples vary.
9. Examples vary.

NAVIGATION -- ANSWER KEY

1. Latitude, 40° N; longitude, 70° W
2. Latitude, 60° S; longitude, 50° E
3. Latitude, 80° N; longitude, 80° N
4. Latitude, 20° S; longitude, 30° E
5. Latitude, 80° S; longitude, 80° W
6. C and E
7. - 10. See below.



11. 90° T
12. 135° T
13. 60 miles in one hour; 20°
14. a. 110° T
    b. Resultant ground speed = 170
15. Approximate 220 mph
16. 90 miles in one hour, approximately 25°
17. 115° T

PAINTING AND PAPERHANGING -- ANSWER KEY

1. 707 square feet
2. Approximately $3\frac{1}{2}$ gallons = 3.535 gallons
3. a. $32 \sqrt{1440} = 1216$ square feet
   b. 2.13 gallons
4. a. 38.016 coats (Use $\frac{22}{7} = \pi$.)
   b. $190.08
5. $59.76
6. a. $514\frac{2}{3}$ square feet
   b. $83\frac{1}{3}$ square feet
   c. 5.528 rolls = 6 rolls
   d. At $6.95 roll = $41.70
7. 12.365 gallons
   height of gable end is $\frac{26}{4} = 6\frac{1}{2}$ feet
8. 625.04 square feet
9. 3.6 gallons
10. $53\frac{1}{3}$ square feet

LANDSCAPE TECHNOLOGY -- ANSWER KEY

1. a. 24% nitrogen, 16% phosphorus, 12% potash

   b. 27,000 squre feet

   c. 151.87

2. 12.5 cups

3. a. 8.07 tons

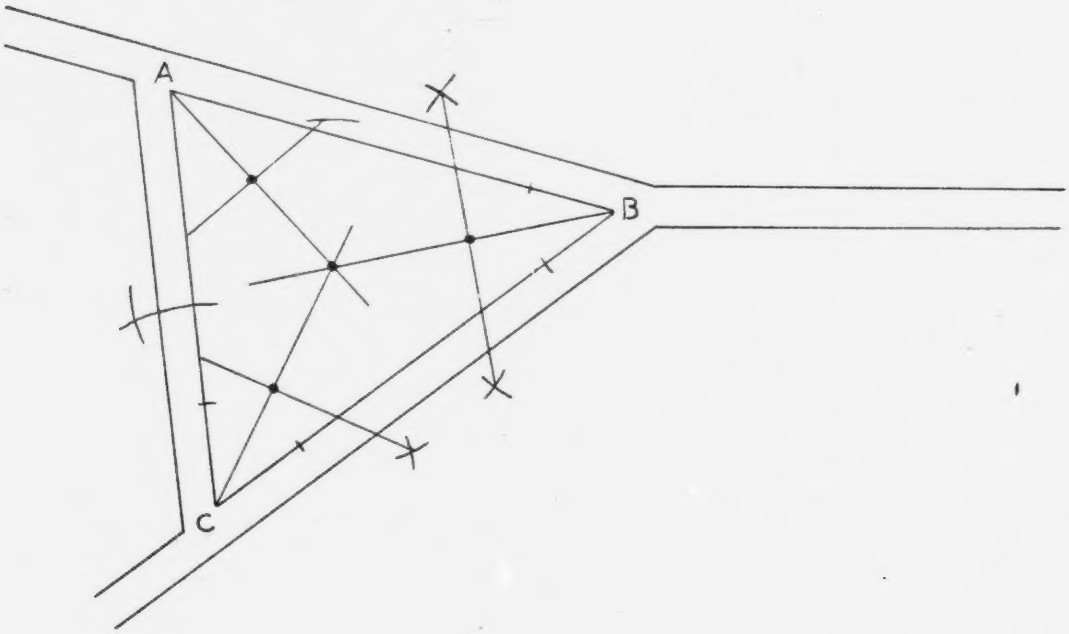
   b. 254 feet

   c. 130.68 cubic yards

   d. 35.64 pounds

4. a. Construct angle bisectors for two angles.

   b. Construct the ⊥ bisector of each angle bisector.



5. $1,248.42

6. 310.86 square feet

7. 207.84 square feet

8. 117.15 cubic feet or 4.34 cubic yards

CARPENTER -- ANSWER KEY

1. 60.8' and 60.8'

2. 5'

3. 10'

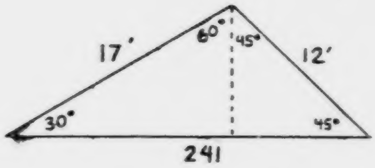
4. 20' approximate

5. 21' 6"

6. 468 square feet 15 sheets of 4' by 8' plywood

7. 70.3 square feet

8. 17 and 12 approximate



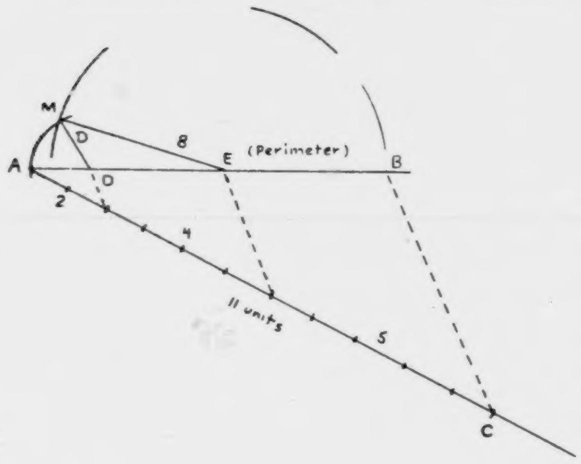
9. 6 squares

10. 160 cubic yard

11. 884 cubic feet

ARCHITECTURE AND DRAFTING -- ANSWER KEY

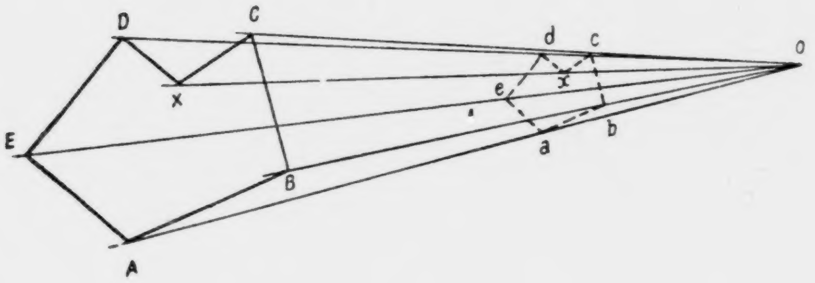
1. Sketch



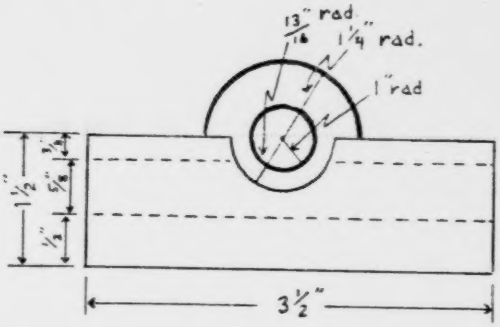
Draw AB 8 inches long.
Divide AB with parallel lines into parts of ratio $\frac{2}{11}, \frac{4}{11}, \frac{5}{11}$.
At point D, with arc of radius AD, swing an arc about C.
At point E, with arc of radius EB, swing an arc about E.
Intersection of these arcs call M.
Required triangle is MDE.

2. Hints: 1. Sum of acute angles is 90°
   2. Hypotenuse of right triangle is the diameter of the circle that circumscribes it.

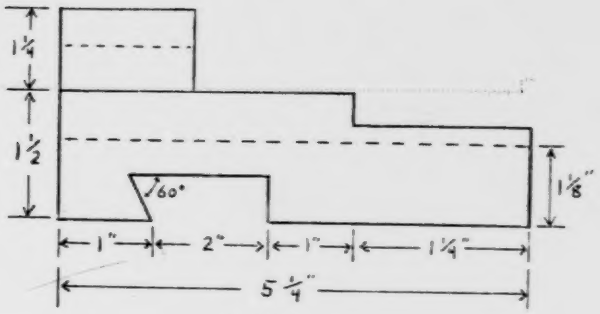
3. Follow directions in problem carefully.



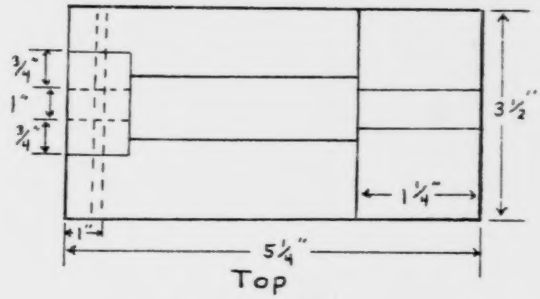
4.



Front



Side



Top

The above are sketches of elevations. Plan should be laid out as shown on page 5 at bottom.

5. Approximately 5.25 or $5\frac{1}{4}$ inches.

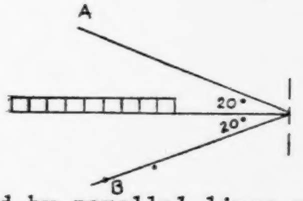
6. a. Approximately 3.5^{o}
   b. Approximately 0.0847 inches

7. 25" x $16\frac{7}{8}$"

8. 7 inches

9. Plan should be 5" by $4\frac{3}{16}$"

OPTICAL TECHNICIAN -- ANSWER KEY



1. 35°

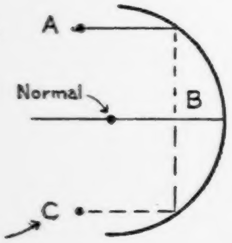
2. 20°

3. Yes
corresponding ∠ 's formed by parallel lines are ≅

4. 60°

5. Inward

6.



90° =Angle of reflection

7.

8. a. RTBA

b. OƵBA

c. ORA

d. ƵTB

e. ƵORT

9. ORT Ƶ

10. ORA or ƵTB

SHEET METAL -- ANSWER KEY

1. a. 31.4 square feet (Use = 3.14.)

   b. 4 sheets

2. a. d = 2 408.7 inches = 40.4 inches

   b. A = 81.25 square feet

3. a. 70.65 cubic feet

   b. = w = h = 4.14 feet

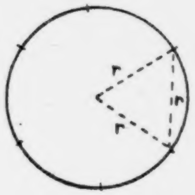
4. $\frac{555}{8}$ square feet = 69.375 square feet

5. 25° and 65°

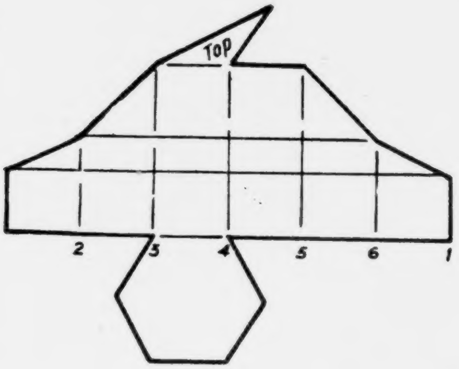
6. 4.71 inches

7. The length of one side

8. Students' diagram should show dividing the circle into six equal parts. Construct this by walking the compass (opened to length of circle radius) around the circumference. Now find centers of holes to be drilled.



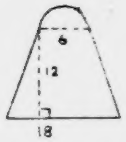
9.



ENGINEERING -- ANSWER KEY

1. 22.11 square feet

2. a.



b. $195\frac{5}{9}$ cubic yards

3. 2781.84 pounds

4. 2563.22 square rods = 77,537.405 square yards

5. b. Area = 10,000 square yards

For the map, follow the hint on page 5.

6. $20\sqrt{2}$ feet = 28.20 feet

7. a. Approximately 620 feet

b. They are perpendicular ($\overline{OT} \perp \overline{TP}$).

8. 868.05 cubic feet

9. a. 8 feet

b. 4 feet

c. 12.8 feet

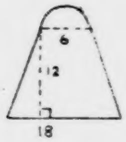
d. 10.8 feet

e. 32.3 feet

ENGINEERING -- ANSWER KEY

1. 22.11 square feet

2. a.



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d. 10.8 feet

e. 32.3 feet

MACHINIST -- ANSWER KEY

1. 60

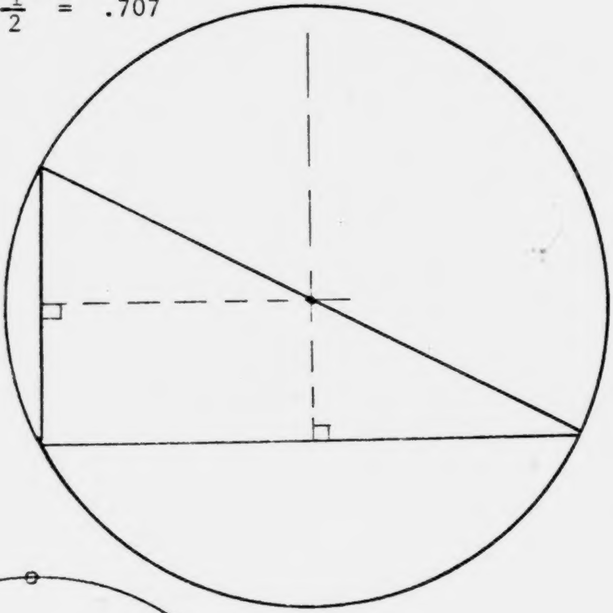
2. $\sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}} = .707$

3. $\sqrt{18}$

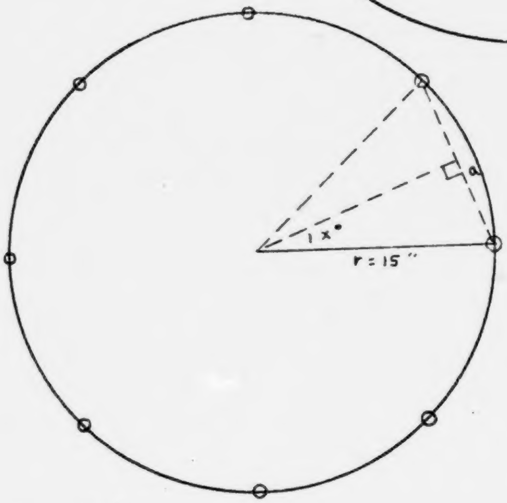
4. a. .8660

   b. 2.3"

   c.



5.



Step 1. 45°

Step 2. $22\frac{1}{2}^{\circ}$

Step 3. Sine $22\frac{1}{2}^{\circ} \doteq .3826$

Step 4. $a \doteq 1.913$

Step 5. $2a \doteq 3.826$

Step 6. Sine $45 = \frac{1}{A}$

$.7071 = \frac{1}{A}$

$.7071A = 1$

$A = \frac{1}{.7071} = 1.414$

CEMENT WORKER -- ANSWER KEY

1. $m\angle A = 60$

2. 8'

3. $A = \frac{1}{2}bh$ (or $A = \sqrt{s(s-a)(s-b)(s-c)}$) $= \frac{1}{2} \times 8 \times 6.928 = 27.7$ square feet

4. Total area = 166.27

5. $166.27 \times .333 = 55.4$ or $55\frac{1}{2}$ cubic feet

6. 27 cubic feet = 1 cubic yard $\qquad 2.05 \doteq 2$ cubic yard

7. a. Total side area = 88 square inches + 176 square inches + 264 square inches + 1248 square inches = 1776 square inches

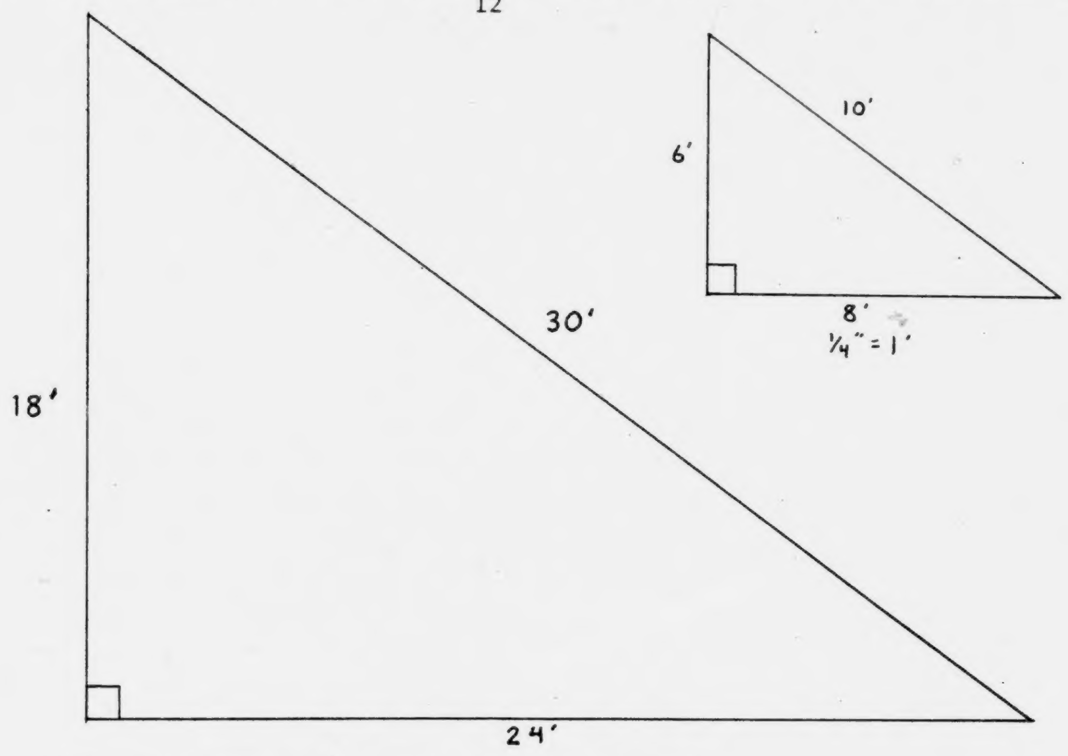
   b. $12\frac{1}{3}$ square feet

   c. $V = 12\frac{1}{3}$ square feet x 4 feet $= 49\frac{1}{3}$ cubic feet

   d. $49\frac{1}{3} - 16$

   $49.33 - 16.4 = 32.9 \doteq 33$ cubic feet = 1.22 cubic yards

8. 5358 square feet $\quad V = 5358 \times \frac{4}{12} = 1784.2$ cubic feet $\quad V = 66.08$ cubic yards



9. Ratios of corresponding sides are in proportion

CEMENT WORKER -- ANSWER KEY

10. 1:3

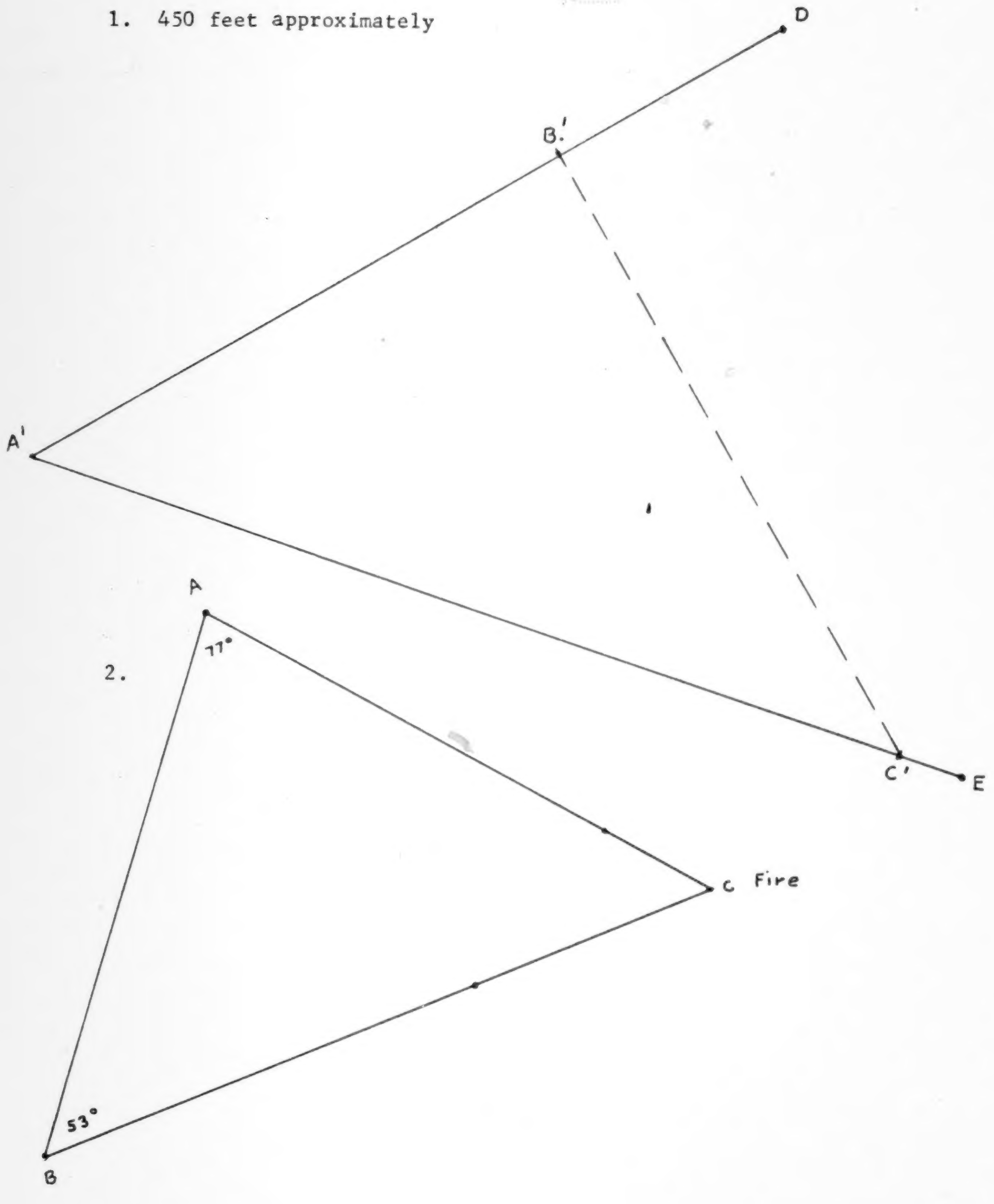
11. 1:9 (smaller △'s A = 24 square feet, larger A = 216)

12. 9 times

13. The areas vary as the square of the ratio of the sides

FORESTRY -- ANSWER KEY

1. 450 feet approximately

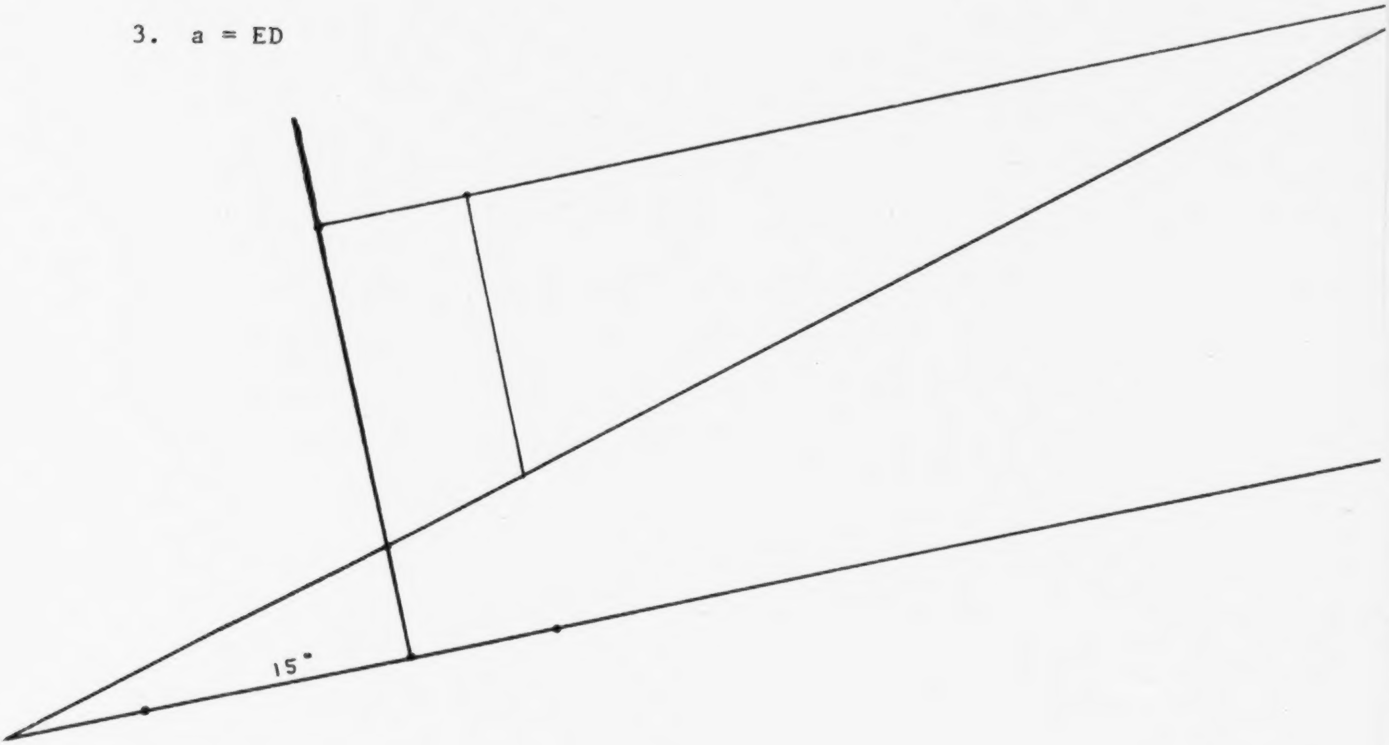


2.

a = 6.1 miles
b = 7.6 miles

FORESTRY -- ANSWER KEY

3. a = ED



b = 114 approximately

4. 20.73 feet

5. 53.98 inches

6. a. 90°

   b. the distances are equal

   c. distance DC

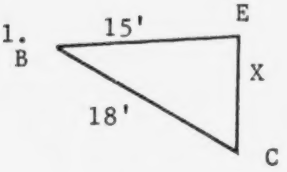
7. a. 110°

   b. 225°

   c. 310°

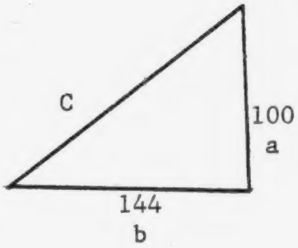
ELECTRICIAN -- ANSWER KEY

1.



$$18^2 = 15^2 + X^2$$
$$18^2 - 15^2 = X^2$$
$$324 - 225 = X^2$$
$$99 = X^2$$
$$10 = X$$

2.



$$c^2 = 100^2 + 144^2$$
$$c^2 = 10{,}000 + 20736$$
$$c^2 = 30{,}736$$
$$c = \sqrt{30{,}736}$$
$$c = 175$$

3. a. $C = 3.14 \times 7 = 21.98''$

b. $m\,\overset{\frown}{DC} = \frac{115^\circ}{360^\circ}$ of $C = .32 \times 21.98 = 7.03''$

c. Total from A to B = $8\frac{1}{2}'' + 7.03'' + 8'' = 23.53''$

GENERAL CONTRACTOR -- ANSWER KEY

1. $201.375 = $201.38

2. $270

3. a. $1338\frac{3}{4}$ square feet

   b. 41 bundles

   c. 42 bundles

   d. $210

4. Approximately 10.88 cubic yards

5. 6.72 hours

   $45.70

6. a. 54'-11"

   b. 177'-10"

   c. 14 + 1 = 15 joists

   d. $90.60

7. Area = 2244 square rods

   = 14 acres

   Cost = $42,000

8. Gravel = 75 cubic yards

   Sand = $56\frac{1}{4}$ cubic yards

   Cement = $18\frac{3}{4}$ cubic yards

9. 62.8 cubic feet = 2.326 cubic yards

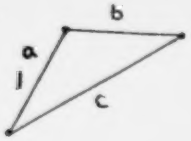
   Cost = $48.85

10. $19\frac{1}{4}$ square feet (Use $\pi = \frac{22}{7}$.)

11. 606 square feet

HOME PLANNING -- ANSWER KEY

1.

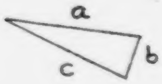


C. a = 5 feet b = 5 feet c = 9 feet

D. P = 19 feet

E. S = 9.5 A = 9.8 square feet

2.

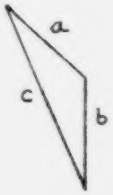


C. a = 6 feet b = 2 feet c = 6 feet

D. P = 14 feet

E. S = 7 A = 5.9 square feet

3.



C. a = 5' b = 5' c = 9'

D. P = 19'

E. S = 9.5 A = 9.8 square feet

4. A. P = 17 feet
   B. A = approximately 12 square feet

5. A. P = 15
   B. A = approximately 10 square feet

6. A. P = 12
   B. A = 6 square feet

HOME PLANNING -- ANSWER KEY

7. Sample $\frac{3}{8}$"

8. $1\frac{5}{8}$"

9. $\frac{3}{8}$"

10. $\frac{2}{8}$"

11. $\frac{5}{8}$"

12. $\frac{2}{8}$"

13. $\frac{5}{16}$"

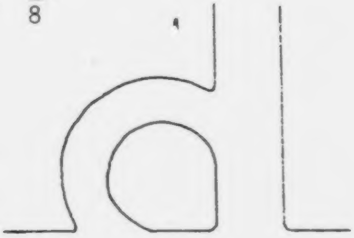
14. $\frac{3}{8}$"

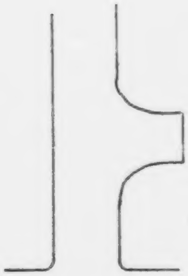
15. $1\frac{5}{8}$"

16. $\frac{3}{16}$"

17. $\frac{3}{16}$"

18. $\frac{2}{8}$"





19. 78'

20. 18'

21. 1404 square feet

22. 30'

23. 2826 square feet

24. 18'

25. 1017 square feet

26. 1809 square feet

27. 3213 square feet

28. 1070 cubic feet

29. 40 cubic yards

30. $840

31. $430

32. Driveway B

CABINETMAKING -- ANSWER KEY

1. e. A = $6\frac{3}{4}$"

B = $1\frac{1}{4}$"

C = $\frac{3}{4}$"

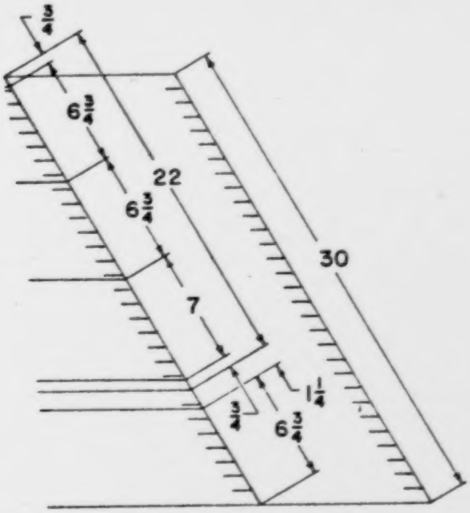
D = 7"

E = $6\frac{3}{4}$"

F = $6\frac{3}{4}$"

G = $\frac{3}{4}$ "

H = $23\frac{1}{4}$"

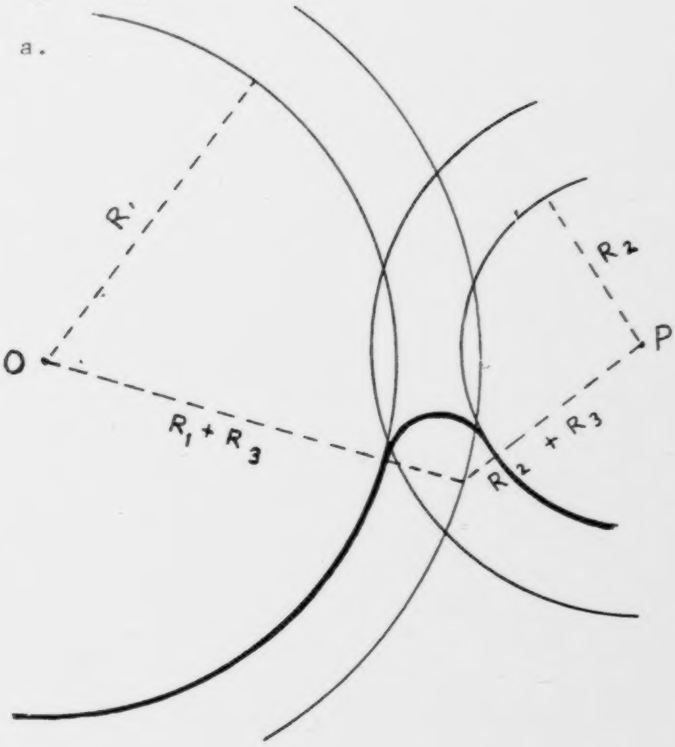


2. a. 40"

b. 20"

3. $\frac{135}{2} = 67\frac{1}{2}^{\circ}$

4. a.



Let $R_1 = 2$

$R_2 = 1$

$R_3 = \frac{1}{2}$

b. Let R_1 be radius of $\widehat{CD}$

Use K as center and draw arc with radius $R_1 + 1$

Construct line parallel to $\overline{AB}$, 1" above $\overline{AB}$

Where parallel line and arc of radius $R_1 + 1$ intersect locates unkown arc's center.

5. 169.56 inches

6. .86 square feet

7. Use a large sheet of paper or cardboard. Need 13" x 28" for end view.

8. $6\frac{1}{4}$ inches

9. The $\perp$ bisectors of two chords intersect at the center of the circle.

PLUMBING AND PIPE FITTING -- ANSWER KEY

1. 36

2. 16

3. 20

4. Radius of single pipe = $\frac{61}{16}$ = 3.81

   Diameter of single pipe = 7.61

5. Examples will vary. Doubling the diameter will increase capacity four times.

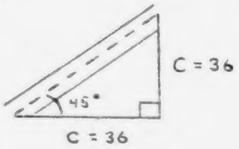
6. Area of piston = $\pi 4^2 = 16\pi$ = 50.24 square inches

   Total pressure = 50.24 x 80 = 4019.2

7. a. 45°

   b. $5\sqrt{2}$

8. $36\sqrt{2}$



9. 10

10. $10\sqrt{2}$

SURVEYOR -- ANSWER KEY

1. N 50° W

2. S 68° W

3. N 30° W

4. S 70° E

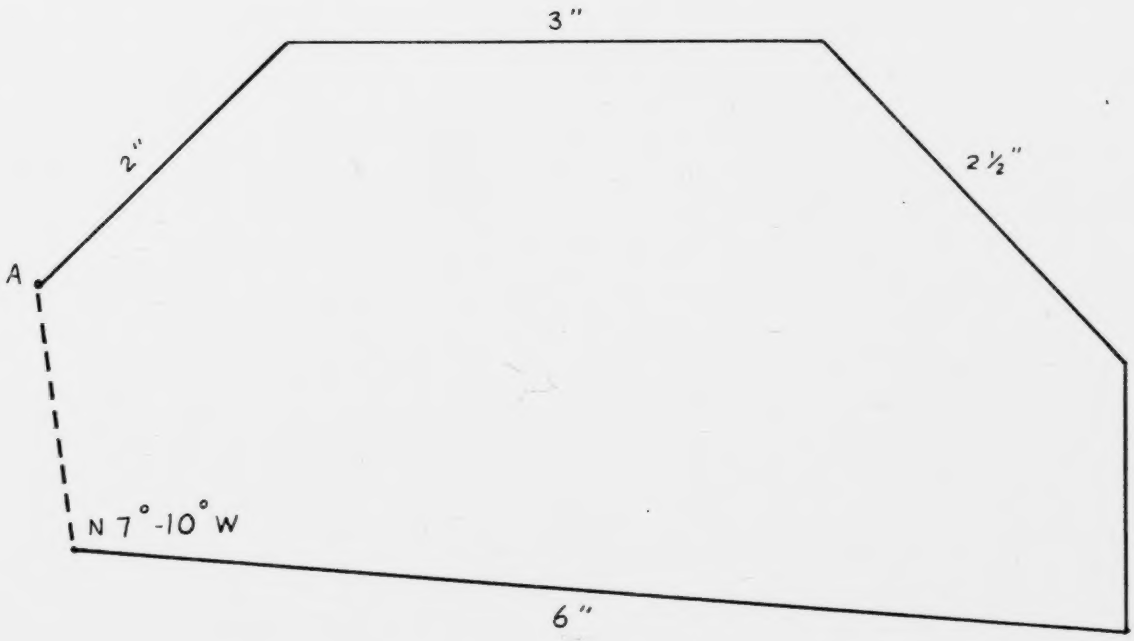
5. a. 15" x 6"

   b. h = 25', w = 20'

   c. AB = 27', BC = 39'

      CD = 39' DA = 36'

6.



   a. 75'-$1\frac{1}{2}$"

   b. N 7° -10° W (Allow for some error)

7. From point A N 80° E 100'; N 90° E 150'; S 50°E 125'; S 0° 75'; N 90° W 200'; N 35° W 62.5'

8. 3" = 150'

9. N 60° W

OUTDOOR ADVERTISING -- ANSWER KEY

1. $3\frac{3}{7}$ (3.428)

2. $13\frac{1}{2}$"

3. 18"

4. 27"

5. 36"

6. 45"

7. 8'-11"

8. 19'-6"

9. $2\frac{20}{107}$ (2.187)

10. 9'-7"

11. 21'-7"

12. $2\frac{29}{115}$ (2.252)

13. 10'

14. 22'

15. $2\frac{1}{5}$ (2.200)

16. 672 square feet

17. 872 square feet

18. a. 1 X 3 (nearest $\frac{1}{4}$)
    b. Bulletin
    c. Embellished

19. a. 1 X $2\frac{1}{4}$ (nearst $\frac{1}{4}$)
    b. Poster
    c. 30-sheet poster

20. a. 1 X $2\frac{1}{4}$ (nearest $\frac{1}{4}$)
    b. Poster
    c. 30-sheet poster

21. a. 1 X $2\frac{1}{4}$ (nearest $\frac{1}{4}$)
    b. Poster
    c. Bleed poster

22. a. 1 X $2\frac{1}{4}$ (nearest $\frac{1}{4}$)
    b. Poster
    c. Bleed poster

23. a. 1 X $5\frac{1}{4}$ (nearest $\frac{1}{4}$)
    This is one of the obvious exceptions to the main rule.
    b. Bulletin
    c. Embellished

SPACE -- ANSWER KEY

1. a. 1,040

   b. $173 \frac{1}{3}$

   c. 20.02 centimeters

2. a. $\frac{s}{\sqrt{\pi}}$

   b. $\frac{s}{2} \sqrt{\pi \sqrt{3}}$

3. a. $\sqrt{(r+h)^2 - r^2}$

   or

   $\sqrt{h(h+2r)}$

   b. 895.54

   c. 889.94

   d. 895.54

   e. 889.94

   f. .63%

4. a. 395.09

   b. 388.84

   c. 1.58%

5. 1,737.5 kilometers

6. 88.1 days

7. a. 1.23%

   b. 2.03%

   c. 3960 miles

   d. 42.46%

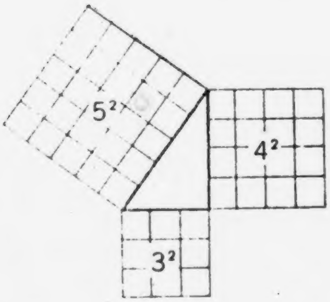
   e. 42.46%

I-1 Right Triangles and the Pythagorean Theorem

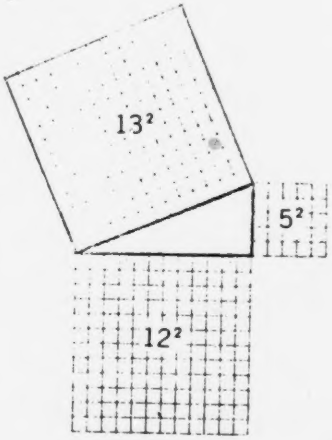
RIGHT TRIANGLES

Many years ago Egyptians used the idea that a triangle with sides 3,4, and 5 units long is a right triangle. They used this fact in construction and surveying. The Babylonians knew other right triangles besides the 3-4-5 triangle. For example, they were aware of the fact that a triangle whose sides were 5, 12, and 13 units long is also a right triangle.

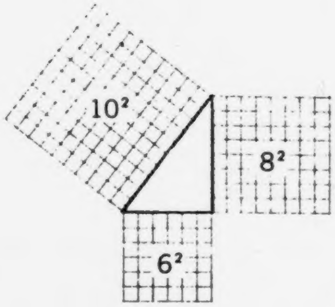
Let's investigate a relation between the lengths of the sides of a right triangle. Physical representations of square regions can be cut from graph paper and arranged so that their edges outline a triangle as shown below.



I.



II



III.

Measure the angles of the triangles above. You should discover that all three triangles have a right angle and therefore are right triangles.

RIGHT TRIANGLES

Consider the lengths 3, 4, and 5 of the sides of triangle I.

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Consider the lengths 5, 12, and 13 of the sides of triangle II.

$5^2 + 12^2 = 13^2$

$25 + 144 = 169$

Consider the lengths 6, 8, and 10 of the sides of triangle III.

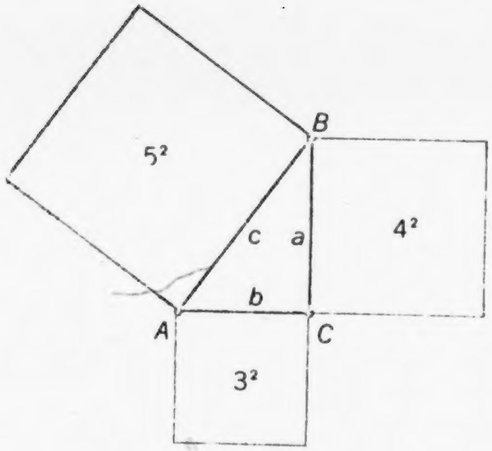
$6^2 + 8^2 = 10^2$

$36 + 64 = 100$

You can see it is possible for certain squares having sides equal in length to a, b, and c and respective areas of a^2, b^2, and c^2 square units to be arranged so as to outline a right triangle if $a^2 + b^2 = c^2$ as shown with the 3-4-5, 5-12-13, and 6-8-10 triangles above.

1. Cut three squares out of a sheet of graph paper having lengths 9, 12, and 15. Arrange them on a sheet of paper so that their edges form a triangle.
    a. What is the measure of the largest angle? ________________
    b. What kind of triangle is formed? ______________
    c. If a, b, and c represent the lengths of the sides of the triangle, is it true that $a^2 + b^2 = c^2$? ____________
    Show this!

Consider once again the 3-4-5 triangle. We know now that $3^2 + 4^2 = 5^2$ and that the triangle is a right triangle. Look again at the geometric interpretation of this fact on the following page.



Notice that the length of the side opposite angle A is represented by the letter a . The length of the side opposite angle B is represented by b , and the length of the side opposite angle C by c.

2. a. Which angle of △ ABC is a right angle? ______________

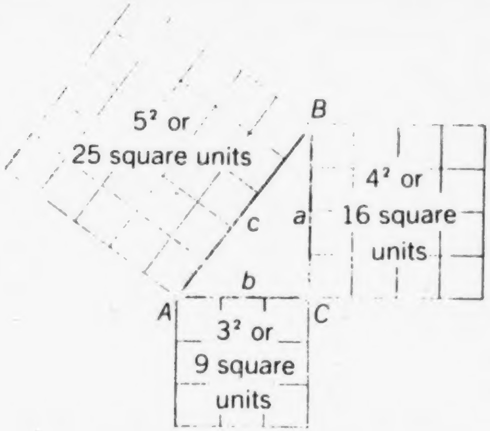
   b. BC and AC are called the "legs" of right triangle ABC. The longest side of a right triangle is called the "hypotenuse." What line segment is the hypotenuse of right triangle ABC?

   ____

   c. The hypotenuse is the side opposite the _____________ angle. The right angle is always opposite the _________________.

The diagram on the following page shows that if each side of the right triangle is one side of a square, then the area of the largest square is equal to the sum of the areas of the two smaller squares.

RIGHT TRIANGLES



We call this relationship $a^2 + b^2 = c^2$ of the sides a, b, and c of any right triangle the Pythagoream Theorem: that is,
"For any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse."

3. The lengths of the sides of two triangles are given below. Are they right triangles?
   a. 9 in., 11 in., 12 in. ______________
   b. 12 ft., 16 ft., 20 ft. ______________

The formula $a^2 + b^2 = c^2$ allows us to find the length of one side of a right triangle when you know the lengths of the other two sides. Suppose the two legs were 33 inches and 44 inches. You could find the length of the hypotenuse c in the following way.

$$a^2 + b^2 = c^2$$

$$33^2 + 44^2 = c^2$$

$$1089 + 1936 = c^2$$

$$3025 = c^2$$

$$\sqrt{3025} = c$$

Remember you are looking for c and not c^2. Therefore, you must find the "square root" of c^2 to find c.

$$c = \sqrt{3025}$$

$$c = 55 \text{ in.}$$

A simple check on your answer is to multiply 55 times 55 and your product should be 3025. Suppose you knew the length of one leg (any one) and the hypotenuse. You could find the length of the other by first subcracting $c^2 - a^2$ or $c^2 - b^2$ and then taking the square root. Going back to the previous problem, if we know one leg is 33 inches and the hypotenuse is 55 inches, we can find the length of the other leg like this:

$$\text{If } a^2 + b^2 = c^2 \text{, then } b^2 = c^2 - a^2$$

$$b^2 = 55^2 - 33^2$$

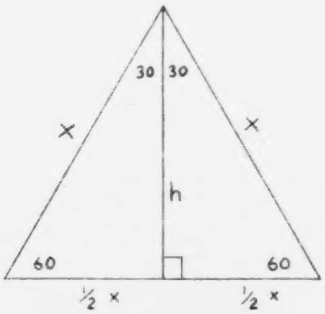
$$b^2 = 3025 - 1089$$

$$b^2 = 1936$$

$$b = \sqrt{1936} = 44 \text{ in.}$$

There are some special right triangles that lend some shortcuts in finding lengths of sides. These are the 30° -60° - right triangle and the 45° -45° - right triangle.

Consider the equilateral triangle below. Each angle is 60° . Now if we draw an altitude as pictured, then two 30-60-right triangles are formed. The altitude also bisects the side of the equilateral triangle that it is perpendicular to.



RIGHT TRIANGLES

If the length of each side of the equilateral triangle is x, then each of the two segments that the altitude divides the side of the triangle into are $\frac{1}{2}x$ as pictured on the pr~~evious~~ page. Now we will find the length of the altitude h by using $a^2 + b^2 = c^2$.

$$h^2 + \left(\frac{1}{2}x\right)^2 = x^2$$

$$h^2 = x^2 - \frac{1}{4}x^2$$

$$h^2 = \frac{4}{4} \cdot x^2 - \frac{1}{4} \cdot x^2$$

$$h^2 = \frac{3}{4} \cdot x^2$$

$$h = \sqrt{\frac{3}{4} \cdot x^2}$$

$$h = \frac{x \cdot \sqrt{3}}{2}$$

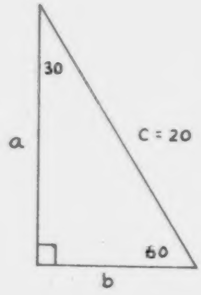
$$h = \frac{1}{2}x \cdot \sqrt{3}$$

To summarize this, if you have a 30°-60°-right triangle, then

a. The length of the shortest leg is one-half the length of the hypotenuse

b. The length of the longest leg is the length of the shortest leg times $\sqrt{3}$.

4. Consider the triangle below.



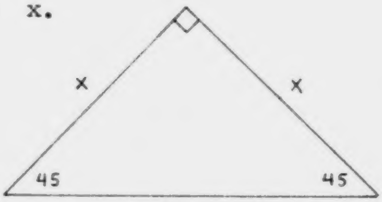
a. If c = 20, then b = $\frac{1}{2}$ · (__________)

b. b = __________________

c. a = __________________

RIGHT TRIANGLES

Suppose we have an isosceles right triangle as pictured below with the lengths of the two equal legs x.

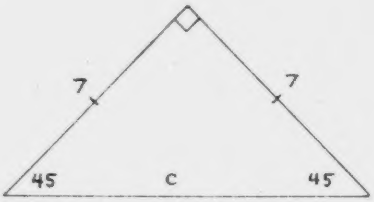


The length of the hypotenuse is found in the following way.

$$x^2 + x^2 = c^2$$
$$2x^2 = c^2$$
$$x \cdot \sqrt{2} = c$$

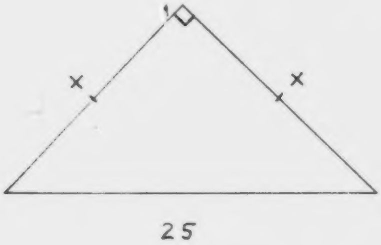
This means that the length of the hypotenuse of a 45°-45°-right triangle can be found by simply taking the length of a leg and multiplying by the $\sqrt{2}$.

5. Consider the triangles below:



a. Find c.

c = ________________________



b. Find x

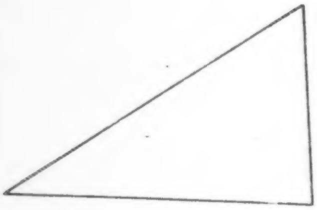
x = ________________________

I-2 Polygons and Their Areas

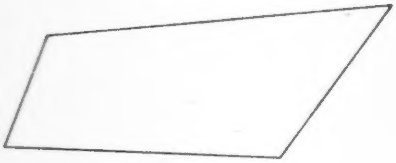
POLYGONS AND THEIR AREAS

Polygons are plane figures formed by line segments which meet at their endpoints. They are named according to the number of sides they have. Some of the common ones are shown below.

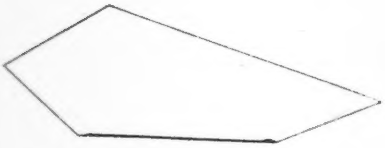
Triangle: 3 sides



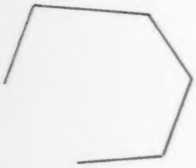
Quadrilateral: 4 sides



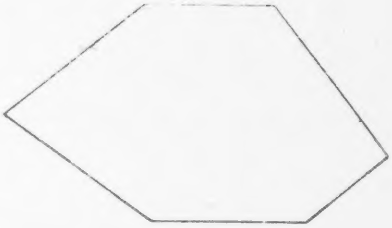
Pentagon: 5 sides



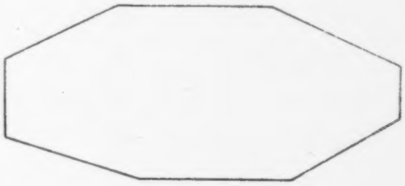
N-gon



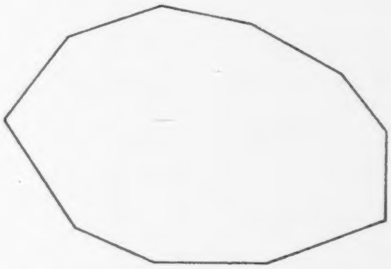
Hexagon: 6 sides



Octagon: 8 sides



Decagon: 10 sides



POLYGONS AND THEIR AREAS

Let's try to develop an easy rule to find the total number of degrees contained in the sum of the measures of all the angles in any polygon. A basic premise we work with is that every triangle has a sum of 180°. With this knowledge, divide each of the polygons pictured on page one into as many distinct triangles as possible. With each, work from one vertex only. Fill in the table below with information gained from the above activity.

Type of Polygon	Sides	△'s Formed	Degrees in Each △	Total Degrees in the Polygon
Triangle				
Quadrilateral				
Pentagon				
Hexagon	6	4	180	4 x 180° or 720°
Octagon				
Decagon				
N-gon	N		180	

To be able to fill in the last row for N-gon we must see the relationship between the number of sides and the number of triangles formed in each polygon. In each, the number of sides minus the number of triangles formed is ____________________. Therefore, the number of triangles formed by the N-gon would be N - 2. The total number of degrees in the N-gon would be: number of △'s formed times number of degrees in each △ or (N-2) X (180).

POLYGONS AND THEIR AREAS

This formula can be used to find the number of degrees in any polygon by substituting the number of sides for N.

1. Find the number of degrees in a seven-sided polygon (heptagon). __________degrees.

A regular polygon is one that has equal length sides (equilateral) and equal measure angles (equiangular). Draw a regular quadrilateral. Give the common name for this figure.

2. Knowing how to find the number of degrees in any polygon and knowing the definition of regular polygons, try to find solutions to the following questions.

   a. Find the number of degrees for each interior angle of a regular pentagon.

   $\angle$ = ______________°

   b. Find the number of degrees for each interior angle of a regular octagon

   $\angle$ = ______________________°

   The formula to find the number of degrees in an interior angle of any polygon is $\frac{(n-2)180}{n} = \frac{\text{sum of all angles}}{\text{number of angles}}$

POLYGONS AND THEIR AREAS

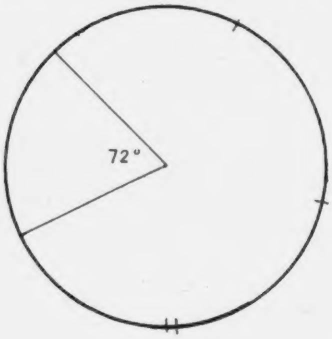
Relating Regular Polygons to Circles

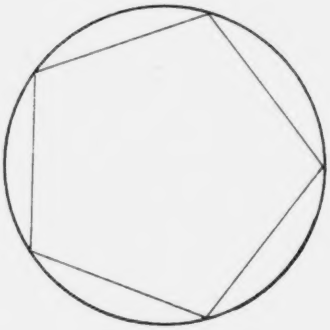
To construct any regular polygon we may make use of the circle. For example, construct a regular pentagon utilizing a circle. Use a compass and protractor for the constructions.

Step 1. Construct the circle (See figure below.)

Step 2. Divide the circle into five (pentagon) equal arcs. Each would have a central angle of $5\overline{)360^{\circ}}$ or 72°. (A central angle has its vertex at the center of the circle.)

Step 3. Connect the five points.





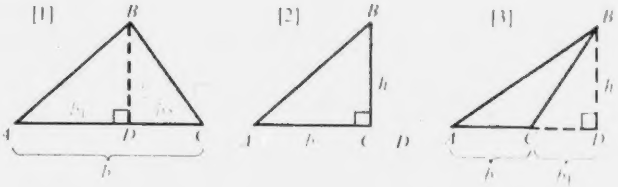
3. What measure of central angle would be utilized to construct an octagon?________
Construct an octagon using the circle method.

POLYGONS AND THEIR AREAS

Finding Areas of Polygons

Because all polygons can be divided into a certain number of distinct triangles, we can find the area of any polygon by finding the sum of the area of these triangles.

Formula for the area of a triangle



$A = \frac{1}{2}bh$

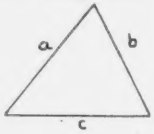
b = base

h = height to base b

Formula for area of a triangle knowing the lengths of the three sides.

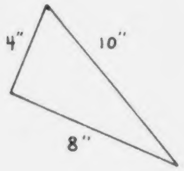
$A = \sqrt{S(S-a)(S-b)(S-c)}$

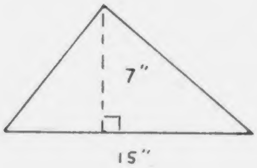
$S = \frac{1}{2}(a+b+c)$



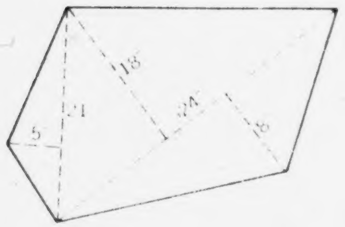
4. Area = ______________________

5. Area = ______________________





POLYGONS AND THEIR AREAS

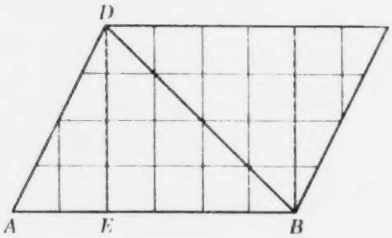


To find the area of the irregular pentagon to the right, we can divide it into triangles whose areas we can find.

6. Find the area of the irregular figure to the right.

Area = ______________________

Common Polygons That Have Specific Formulas for Their Areas

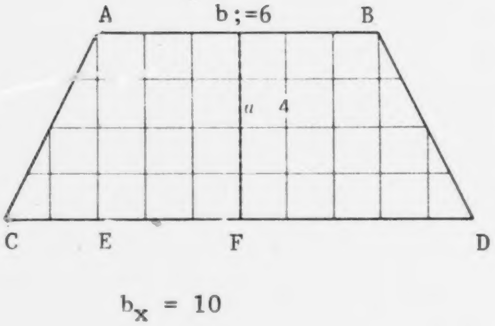


Parallelogram (two pair of parallel sides)

$A = b \times h$

$b = AB$

$h = DE$



Trapezoid (only one pair of parallel sides)

$A = \frac{1}{2} (b_1 \times b_2)h$

b_1 = length of top base

b_2 = length of bottom base

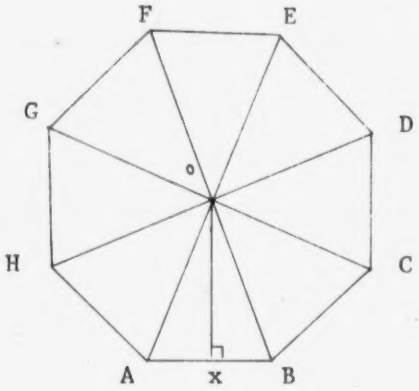
$h = \perp$ height $= AE$

7. Find the area of the trapezoid given above.

Area = ______________________

POLOYGONS AND THEIR AREAS

The area of any regular polygon can be found by finding the area of one of the triangles, then multiplying by the number of sides. In this case the height of △ABO is OX.



Area of a regular polygon = N ($\frac{1}{2}$ bh)

Where

N = number of triangles

b = base of one △ = AB

h = height of one △ = OX

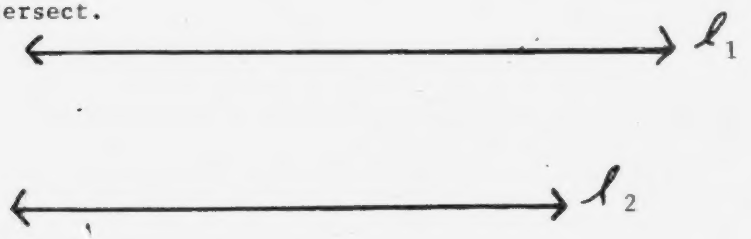
8. Find the area of the regular octagon where OX = 10 and AB = 8.

Area = ______________________________

I-3 Parallel Lines

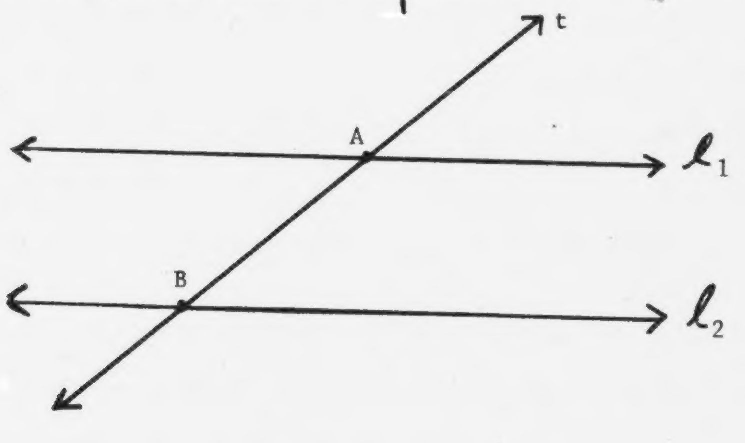
PARALLEL LINES

Definition: Two lines are parallel if they lie on the same plane and do not intersect.



ℓ_1 is parallel to ℓ_2

Many problems exist involving parallel lines which are intersected by a third line. We call this line that intersects the two parallel lines and is in the same plane as the two parallel lines a "transversal."

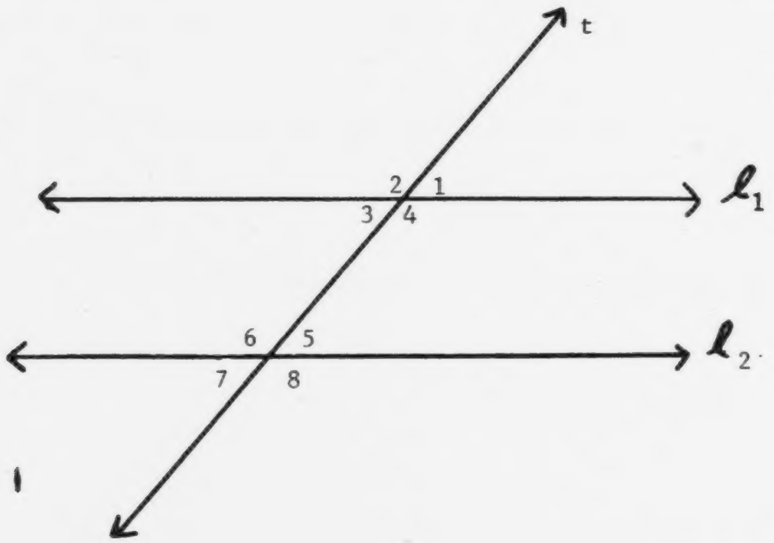


In the above figure, line t is a transversal cutting ℓ_1 and ℓ_2 at the two points A and B. The transversal must cut the two lines in two points (or three lines in three points, etc.).

Keep in mind that a transversal can intersect any two or more lines; however, in this discussion we are only concerned with what happens if we intersect parallel lines with a transversal.

PARALLEL LINES

1. Consider the following parallel lines cut by a transversal



$\ell_1 \parallel \ell_2$
t transversal

Use your protractor to find:

a. m ∠ 1 = ______________
b. m ∠ 2 = ______________
c. m ∠ 3 = ______________
d. m ∠ 4 = ______________
e. m ∠ 5 = ______________
f. m ∠ 6 = ______________
g. m ∠ 7 = ______________
h. m ∠ 8 = ______________

The angles named in the figure above are given "special names in pairs."

2. The pair ∠ 3 and ∠ 5 are called "alternate interior" angles. There is one more pair of alternate interior angles. What is it? a.____________________ and b.________________ How do the measures of a pair of alternate interior angles compare? c.________________________________

3. The pair ∠ 1 and ∠ 5 are called "corresponding" angles. Another pair of corresponding angles is ∠ 4 and ∠ 8. There are two more pairs of corresponding angles. What are they? a.______________and b.______________; c.____________ and d.____________________. How do the measures of a pair of corresponding angl compare? e.________________________

4. Angles 4 and 5 are supplementary (i.e., m $\angle$ 4 + m $\angle$ 5 = 180). Name another similar pair of supplementary angles. a. ______________________________ and b. ____________________________.

There are other important properties of the angles, however the three types mentioned in problems two, three, and four above are the situations we will concern ourselves with mostly in the units to follow. Remember the following facts. Anytime you cut parallel lines with a transversal,

a. The pairs of alternate interior angles have equal measures.

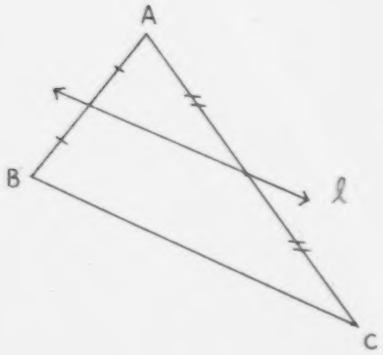
b. The pairs of corresponding angles have equal measures.

c. The interior angles on the same side of the transversal are supplementary.

5. Check your results in problem one with the summary just stated. Do you find this true in problem one? ______________________________ If not, then you better re-measure your angles.

There are some interesting and useful properties concerning parallel lines cut by transversals.

Property 1. If a line is parallel to one side of a triangle and contains the midpoint of a second side, then the line contains the midpoint of the third side.

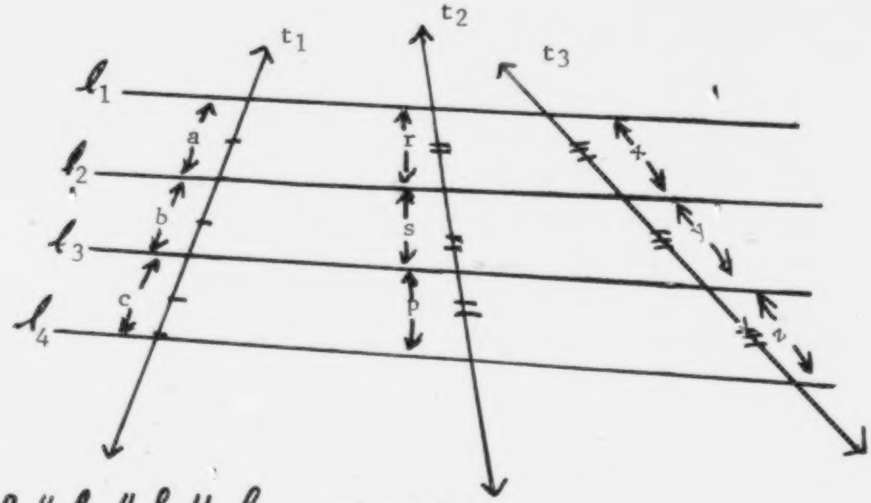


If ℓ contains midpoint of $\overline{AB}$,

ℓ is parallel $\overline{BC}$.

Conclusion: ℓ contains midpoint of $\overline{AC}$

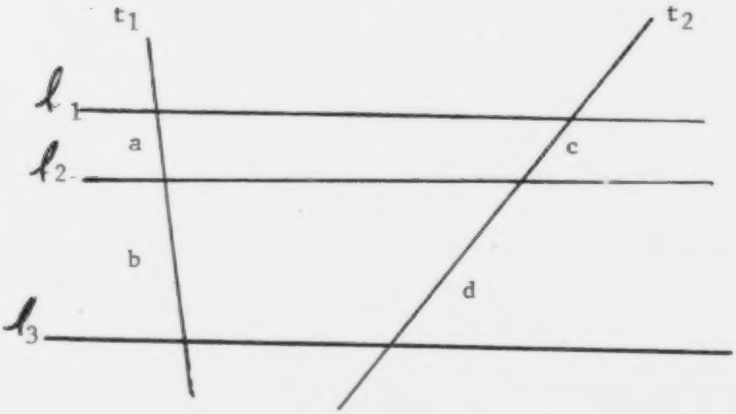
Property 2. If parallel lines cut off congruent segments on one transversal, they will cut off congruent segments on any other transversal intersecting them.



If $\ell_1 \parallel \ell_2 \parallel \ell_3 \parallel \ell_4$ and $a = b = c$

Conclusion: $r = s = p$ and $x = y = z$

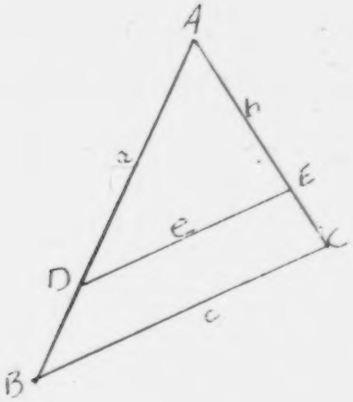
Property 3. If parallel lines cut two or more transversals, then the lengths of the segments intercepted on one transversal are proportional to the lengths of the segments intercepted on the others(s).



If $l_1 \parallel l_2 \parallel l_3$ and t_1 and t_2 are transversals.

Conclusion: $\frac{a}{b} = \frac{c}{d}$ or $\frac{a}{c} = \frac{b}{d}$.

Property 4. If a line intersects two sides of a triangle and is parallel to the third side, then a triangle similar to the original triangle is formed and the ratios of the corresponding sides of the two similar triangles are equal.

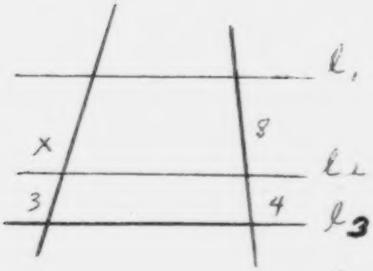


If $\overline{DE} \parallel \overline{BC}$,

Then $\triangle ADE \sim \triangle ABC$ and

$$\frac{a}{AB} = \frac{b}{AC} = \frac{e}{c}$$

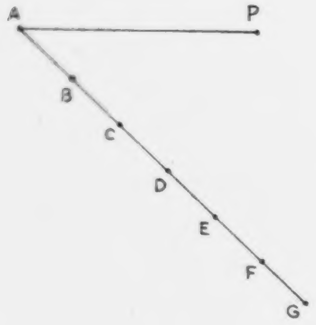
6.



Given $l_1 \parallel l_2 \parallel l_3$

Find x = ______________

7.



If $\overline{AP}$ and $\overline{AG}$ intersect at A, and AB = BC = CD = DE = EF = FG, describe a method for partitioning $\overline{AP}$ into six congruent segments.

Could you now partition $\overline{AB}$ into six congruent segments? Triple the lengths of the picture shown with the same angle and show all lines (lightly) that divide $\overline{AB}$ into six equal segments.

I-4 Standard Constructions

STANDARD CONSTRUCTIONS

In this unit you will have the basic constructions at your fingertips. No proofs of any of these are given. The purpose is rather that the student be able to have a short list of the constructions more commonly used and perhaps a few others.

1. Construct a triangle, given the three sides a, b, and c.

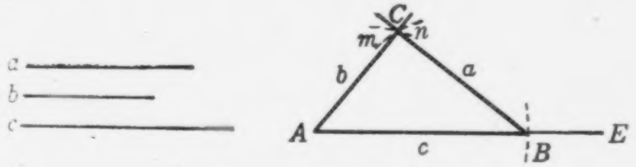
   Procedure

   a. Make AB = c.

   b. Using A as a center and b as a radius, draw an arc.

   c. Using B as a center with radius a, draw another arc which will intersect the previous arc at a point called C.

   d. $\triangle$ ABC is the required triangle.



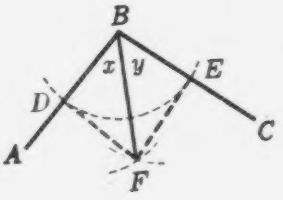
2. Given an angle, construct its bisector.

   Procedure

   a. Using B as a center and any radius, draw an arc which intersects angle sides at D and E.

   b. Using D and E as centers and equal radii, draw arcs which then intersect at F.

   c. $\overline{BF}$ is the required bisector.



STANDARD CONSTRUCTIONS

3. Duplicate an angle (∠ CDE) with a given vertex (A) and side ($\overline{AB}$).

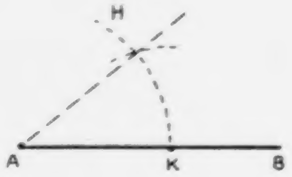
Procedure

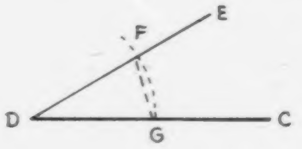
a. Using D and A as centers, draw equal arcs, intersecting ∠'s at F, G, and K respectively.

b. Using FG as a radius and K as a center, draw an arc which will intersect arc $\overline{HK}$ at M.

c. Draw $\overline{AM}$.

d. ∠ MAB is the required angle.





4. Draw a triangle, given two sides and the included angle.

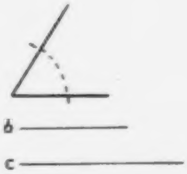
Procedure

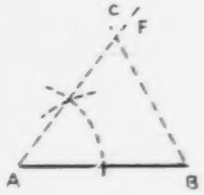
a. Construct $\overline{AB}$ equal to c.

b. At A, construct ∠ BAF = ∠ 1.

c. On $\overline{AF}$, construct AC = b.

d. △ ABC is the required △.



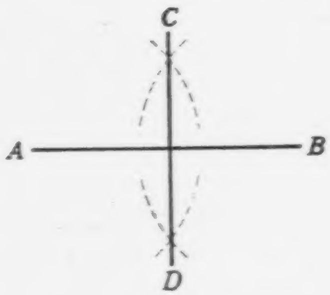


5. Construct the perpendicular bisector of a segment.

Procedure

a. Using A and B as centers and using equal radii, draw arcs intersecting at C and D.

b. $\overline{CD}$ is the required ⊥ - bisector of $\overline{AB}$.



STANDARD CONSTRUCTIONS

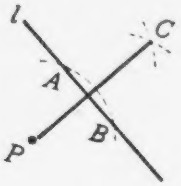
6. Construct a perpendicular to a line from an external point P.

Procedure

a. With P as center, draw an arc which intersects the line at points A and B.

b. Using A and B as centers and equal radii, draw arcs which will intersect at C.

c. $\overline{PC}$ is the required perpendicular.



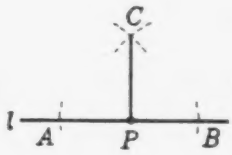
7. Construct a perpendicular to a line at a particular point (P) on that line.

Procedure

a. Using P as center, draw a circle intersecting the line at A and B.

b. With A and B as centers and equal radii, draw arcs which intersect at C.

c. $\overline{PC}$ is the required perpendicular.



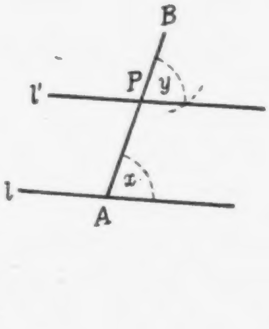
8. Through a given point (P), construct a line parallel to a given line.

Procedure

a. Through P, draw any line which intersects the given line L,

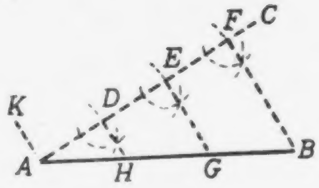
b. Using P as vertex and PB as a side, construct $\angle y = \angle x$.

c. L' is the required line.



STANDARD CONSTRUCTIONS

9. Divide a line segment into any number of equal parts.



Procedure (given for three)

a. From A, draw a line other than $\overleftrightarrow{AB}$

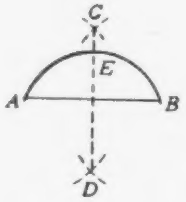
b. Construct three equal segments: AD, DE, and EF, on that line.

c. Draw $\overline{FB}$.

d. Through D and E, construct lines parallel to FB, intersecting at H and G respectively.

e. AH = HG = GB

10. Bisect an arc AB of a circle.



Procedure

a. Draw chord AB.

b. Construct $\overline{CD}$, the perpendicular bisector of $\overline{AB}$, which intersects $\overset{\frown}{AB}$ at E.

c. Then $\overset{\frown}{AE} = \overset{\frown}{EB}$.

11. Find the center of a circle, given only an arc.

Procedure

a. Draw two distinct chords whose endpoints are on the arc.

b. Construct the $\perp$ - bisectors of both chords.

c. Their intersection is the required center.

STANDARD CONSTRUCTIONS

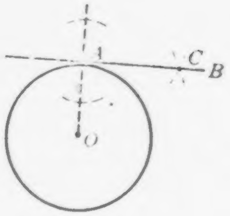
12. Construct a tangent to a circle at a given point (A).

Procedure

a. Draw a line from O through A.

b. Construct a perpendicular to $\overrightarrow{OA}$, through A.

c. This perpendicular is the required tangent.



13. Construct tangents to a given circle from a given external point.

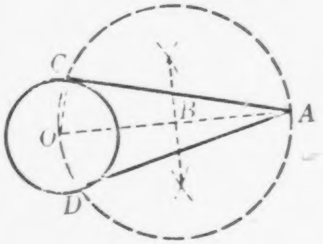
Procedure

a. Draw $\overline{OA}$

b. Bisect $\overline{OA}$, calling that point B.

c. With B as center and OB as radius, draw a circle which intersects the given circle at C and D.

d. $\overleftrightarrow{AC}$ and $\overleftrightarrow{AD}$ are the required tangents.



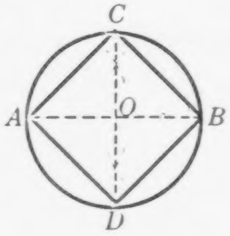
14. Inscribe a square in a given circle.

Procedure

a. Draw any diameter $\overline{AOB}$.

b. Construct a ⊥- bisector of $\overline{AB}$, intersect ⊙ at C and D.

c. ADBC is the required square.



15. Inscribe a regular hexagon in a given circle.

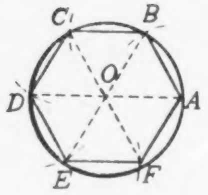
Procedure

a. Draw any radius OA.

b. Using A as center and radius OA, draw an arc which intersects the circle at B.

c. In like manner, using B as center, find C; then D, E, and F.

d. ABCDEF is the required hexagon.

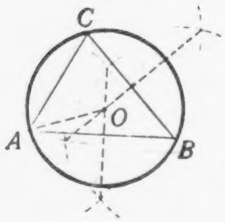


16. Circumscribe a circle about a given triangle.

Procedure

a. Construct ⊥-bisectors of any two sides of the Δ.

b. Using the intersection point as center O, and OA as a radius, construct the required circle.



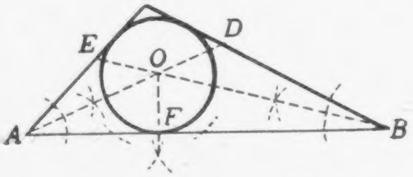
17. Inscribe a circle in a given triangle.

Procedure

a. Construct bisectors of any two angles of the triangle and call the intersection point O.

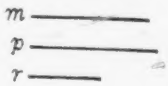
b. From O, construct a perpendicular to side AB and call the intersection point F.

c. Using O as center and OF as the radius, draw the required circle.



STANDARD CONSTRUCTIONS

18. Divide a line segment into n parts which are proportional to n given line segments.



Procedure (shown for three parts)

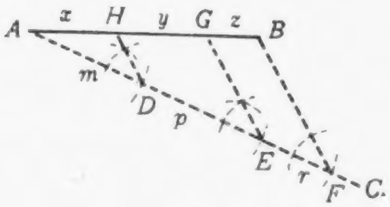
a. Draw $\overleftrightarrow{AC}$, distinct from $\overleftrightarrow{AB}$.

b. On $\overleftrightarrow{AC}$, construct AD = m, DE = P, and EF = r.

c. Draw FB.

d. Through E and D, construct lines parallel to FB, intersecting $\overline{AB}$ at G and H respectively.

e. Thus $\frac{x}{m} = \frac{y}{p} = \frac{z}{r}$



19. Construct a polygon similar to a given one when two corresponding sides are given.

Procedure (ABCDE is given: A'B' is given)

a. From A, draw all diagonals.

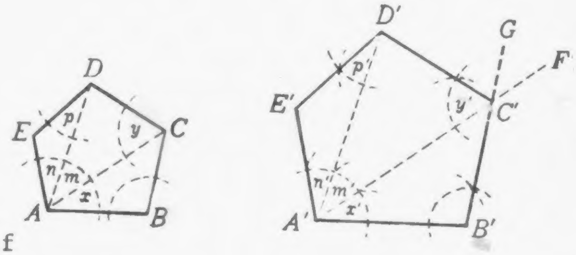
b. At A', construct x' = ∠x and extend side.

c. At B', construct ∠B' = ∠B. The intersection is C'.

d. In like manner, construct ∠m'=∠m, ∠y' = ∠y, and call the intersection of the sides D'.

e. Also in like manner, construct ∠n' = ∠n, ∠p' = ∠p, to find intersection E'.

f. A'B C'D'E' is the required similar polygon.



STANDARD CONSTRUCTIONS

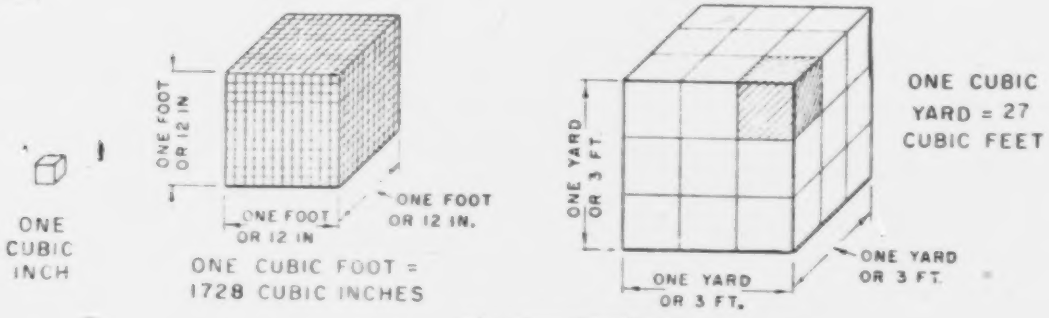
With the basic constructions given here, you should be able to figure out how to construct all of what you would need in the various on-the-job situations. you'll encounter. On separate paper, try the following constructions to see if you've mastered the techniques shown above.

1. Divide the base of a triangle into segments proportional to the other two sides.
2. Through the vertices of a triangle, construct parallels to the opposite sides.
3. Construct a 30° -60° right triangle and inscribe a circle in it. Circumscribe a circle about the triangle.
4. Given a circle and the midpoint of a chord, construct the chord.

I-5 Volume

The cubic yard, which is an application of the volume principle, is used consistently in construction work. The relationships between cubic inches, cubic feet, and cubic yards are shown below.

fig. B



II. Volume of a triangular prism

$V = B \times h$

The base is a right triangle with sides of 3", 4", 5". Find the area of this triangle. (See unit "Polygons.")

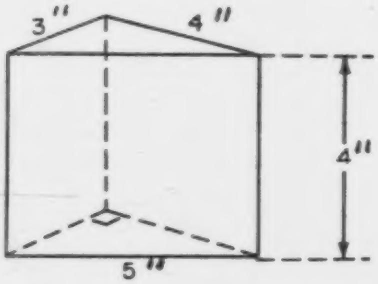
$B = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$ [a, b, c are the lengths of the sides]

$s = \frac{1}{2}(3+4+5) = 6$; $B = \sqrt{6(6-3)(6-4)(6-5)} = \sqrt{6(3)(2)(1)} = \sqrt{36} = 6$ sq. in.

$h = 4$ inches

$V = 6 \times 4 = 24$ cubic inches

fig. C



III. Volume of a pentagonal prism

$V = B \times h$ B = area of the base

Step 1. Find area of the base.

Step 2. Find height.

Solution:

Step 1. Notice the base can be divided into a rectangle and a triangle.

Find the area of the rectangle, then the area of the triangle.

Area of rectangle = $8 \times 10 = 80$ square inches.

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$\text{Where } s = \frac{1}{2}(a+b+c)$$

$$s = \frac{1}{2}(8+8+10) = 13$$

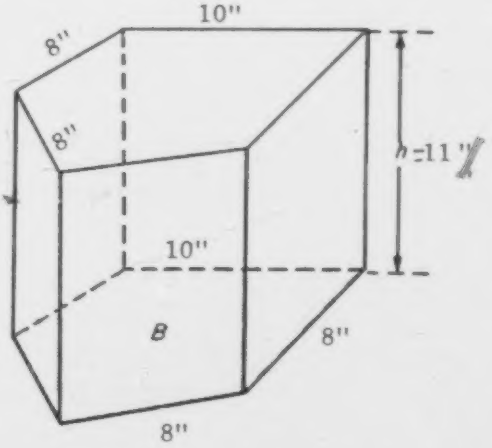
$$\text{Therefore } A = \sqrt{13(13-8)(13-8)(13-10)} = \sqrt{13(5)(5)(3)} = \sqrt{1475} = 38.4$$

Total area of the base = $80 + 38.4 = 118.4$ square inches

Step 2. Height = 4 inches

Volume = $B \times h = 118.4 \times 4 = 473.6$ cubic inches.

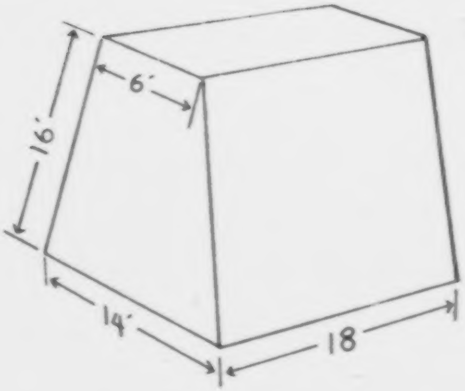
fig. D



VOLUME

IV. Volume of a trapezoidal prism (a prism whose top and bottom bases are trapezoids).

fig. E



Notice the trapezoidal base is not in the bottom position, but on the side; even so, it is, by definition the base of the prism because the opposite side is the same size and shape.

$V = B \times h$

$B = \frac{1}{2}(b_1 + b_2)\,h$

$= \frac{1}{2}(14' + 6')16'$

$= 10.16$

$= 160$ square feet

(See unit, "Polygons," for explanation of area of a trapezoid.)

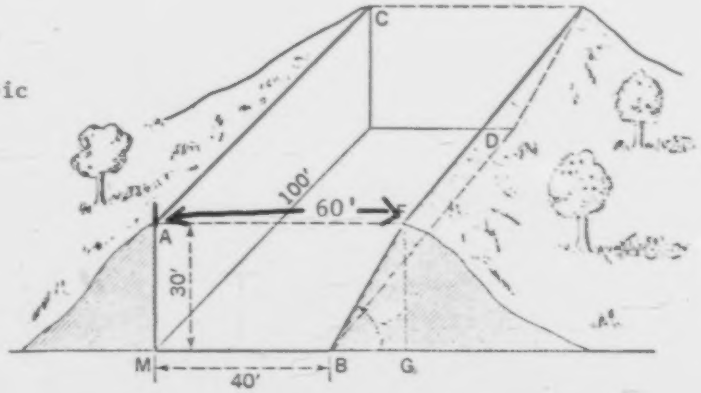
$V = 160.18' = 2880$ cubic feet = a.______________cubic yards. (Find how many cubic yards.)

fig. F

Find the volume of dirt to be removed in figure F. Give your answer in cubic feet and cubic yards.

AF = 60'

MB = 40'



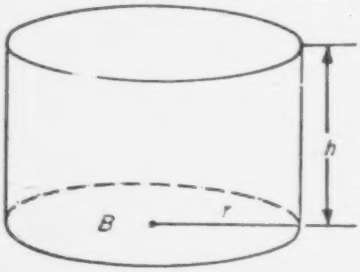
cubic feet = b.______________

cubic yards = c.______________

V. Volume of a circular cylinder

Note: Circular cylinders are not prisms as they do not have polygons for bases.

fig. G



Formula: $V = B \times h$

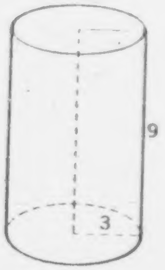
In this case B, the area of the base, is the area of a circle.

$B = \pi r^2$

Example:

Find the volume of this cylinder.

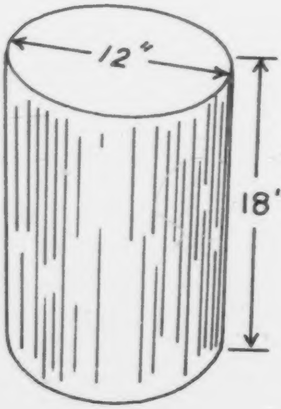
fig. H



$$\begin{aligned} V &= B \times h \\ &= \pi 3^2 \times 9 \\ &= \frac{22}{7} \times \frac{9}{1} \times \frac{9}{1} \\ &= \frac{1782}{7} \\ &= 254.56 \text{ cubic units} \end{aligned}$$

fig. I.

Volume of fig. I = ____________________

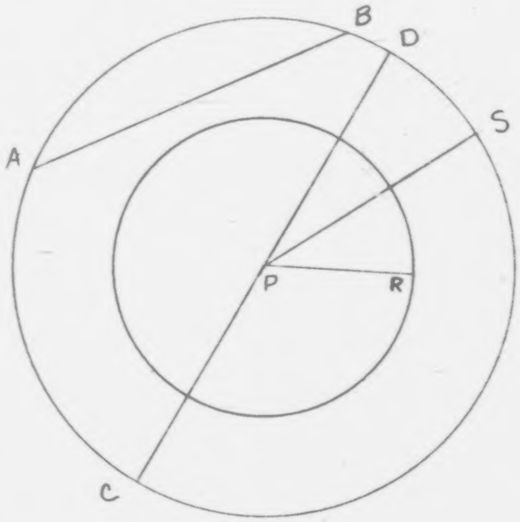


I-6 Circle Relationships

CIRCLE RELATIONSHIPS

For the purposes of this unit, a circle shall be indicated as C(P,r), which will signify a set of points equidistant from a particular point P. P is known as the center and r will be called the radius. Any segment originating at P and having length r may be used as a radius.

Definition: Two or more circles are concentric if and only if they have the same center. In the diagram, circles C(P,R) and C(P,S) are concentric circles.



Definition: A chord of a circle is a segment (i.e., $\overline{AB}$) whose endpoints are points of the circle. A diameter is the measure of the chord which contains the center of the circle; when a plumber talks of a 3-inch pipe, he's referring to diameter.

Definition: Congruent circles are circles with congruent radii.

Fact 1: Radii of congruent circles are congruent.

Fact 2: The diameter of a circle is twice the radius of that circle.

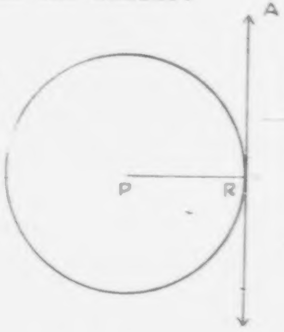
Fact 3: Diameters of congruent circles are congruent.

Definition: The interior of a circle is the set of points in the plane of the circle whose distances from the center are less than the radius.

CIRCLE RELATIONSHIPS

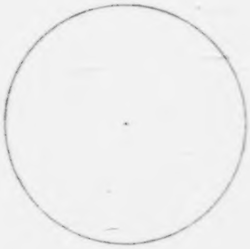
Definition: The exterior of a circle is the set of points in the plane of the circle whose distance from the center are greater than the radius.

Definition: A line coplanar with a circle is tangent to the circle if and only if its intersection with the circle contains exactly one point. For example, RA is tangent to C(P,R) at R, the point of tangency.

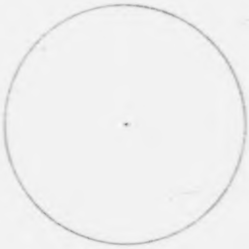


1. In the following three circles, give examples where the intersection of a line and a circle contains 0, 1, and 2 points. Is it possible for the line and circle to intersect at more than two points?

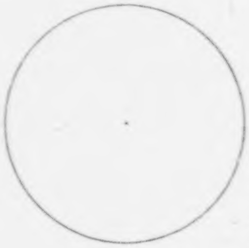
a.



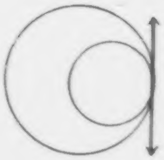
b.

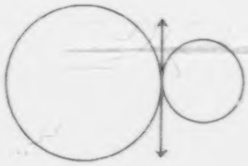


c.



Definition: Two circles are tangent if and only if they are coplanar and are tangent to the same line at the point. Circles may be tangent either internally or externally.





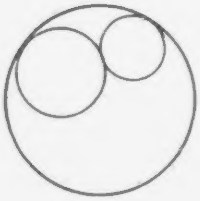
CIRCLE RELATIONSHIPS

Fact 4: A line coplanar with a circle is tangent to the circle if and only if it is perpendicular to the radius whose outer end is the point of contact.

Fact 5: If two circles are tangent, their centers and the point of contact are collinear.

2. Given three circles each tangent to the other two, there are four possible arrangements, one of which is shown. Sketch the other three.

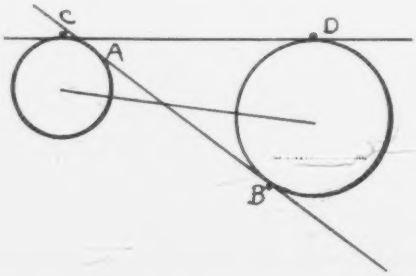
a.   b.   c.



Definition: A line tangent to each of two circles is called common tangent. If the line intersects the segment connecting the two centers, it is called an internal tangent; if not, then it is an external tangent.

$\overleftrightarrow{AB}$ is an internal common tangent

$\overleftrightarrow{CD}$ is an external common tangent



3. Sketch examples showing two circles having 0, 1, 2, 3, and 4 common tangents.

a. "0"

b. "1"

c. "2"

d. "3"

e. "4"

Many facts involving chords and their relations in circles are useful. These will be used later.

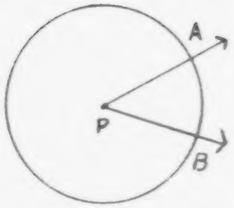
Fact 6: A line containing the center of the circle that bisects a chord other than a diameter, is perpendicular to that chord.

Fact 7: A line containing the center of a circle perpendicular to a chord bisects that chord.

Fact 8: In the plane of a circle, the perpendicular bisector of a chord passes throught the center of the circle.

Fact 9: Chords of congruent circles are congruent if and only if they are equidistant from the centers of their respective circles.

CIRCLE RELATIONSHIPS

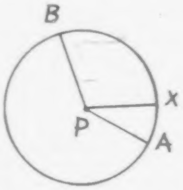


Definition: A central angle is an angle in the plane of the circle which has the center of the circle as its vertex. ∠APB is a central angle.

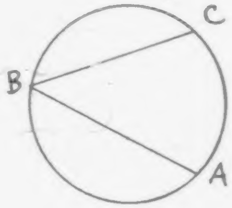
Definition: The measure of any arc shall be the same as the measure of the central angle intercepting that arc.

Fact 10: Arcs of congruent circles are congruent if and only if they have the same measure.

Fact 11: $m\,\widehat{AXB} = m\,\widehat{AX} + \widehat{MXB}$

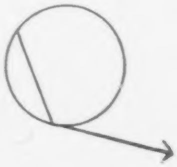


Definition: An angle is inscribed in an arc if, and only if, each side contains one end point of the arc and the vertex of the angle is a point of the arc other than an end point. The arc of the circle whose points lie either in the interior of or on the sides of the inscribed angle is called the inscribed arc.



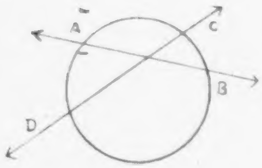
Fact 12: The measure of an inscribed angle is half the measure of its intercepted arc.

Fact 13: The measure of an angle having a chord as one side and a tangent as the other is half the measure of its intercepted arc.



CIRCLE RELATIONSHIPS

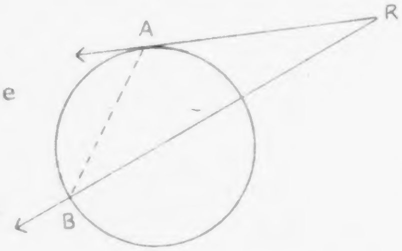
Definition: A secant of a circle is a line which intersects the circle in two distinct points.



Fact 14: The measure of an angle with its vertex in the interior of a circle and whose sides are contained in secants to the circle is half the sum of the measures of its intercepted arc and the arc interdepted by its vertical angle.

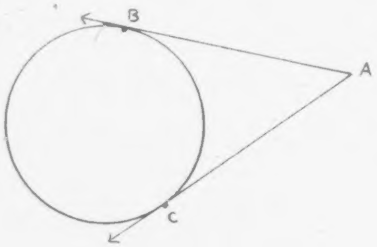
Fact 15: If an angle intercepts two arcs of a circle, its measure is half the difference of the measures of the arcs.

Fact 16: Angles inscribed in the same or congruent arcs of a circle are congruent.



Definition: Given a tangent from external point A to a circle, the tangent segment is the segment with endpoints A and the point of tangency and all points of the tangent between these two.

Fact 17: The two tangent segments from a point in the exterior of a circle are congruent. $\overline{AB} \cong \overline{AC}$

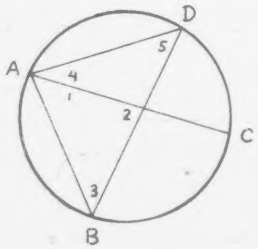


Try the following:

4. If m $\overset{\frown}{AB}$ = 120, m $\overset{\frown}{BC}$ = 90, and m $\overset{\frown}{CD}$ = 60, then:

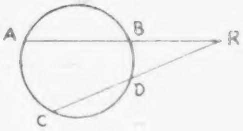
a. m $\angle 1$ = __________ c. m $\angle 3$ = __________

b. m $\angle 2$ = __________ d. m $\angle 4$ = __________



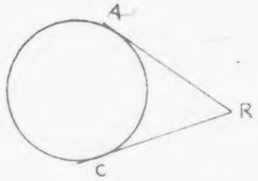
5. If m $\angle 1 = 40$ and m $\angle 4 = 25$, then

a. m $\overset{\frown}{BC}$ = ______________ b. m $\overset{\frown}{CD}$ = ______________



6. If m $\overset{\frown}{AC} = 110$, and m $\overset{\frown}{DB} = 36$, then

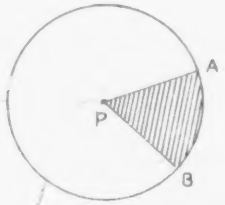
m $\angle$ ARC = ______________



7. If m $\overset{\frown}{CA} = 110$, then

m $\angle$ ARC = ________________

Fact 18: The length ℓ of an arc of radius r and measure m, is $\ell = \frac{m}{180}(\pi r)$

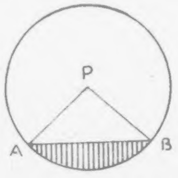


Definition: A sector of a circle is a region bounded by two radii and an arc as shown in the diagram.

Fact 19: The area of a sector of a circle is half the product of its radius and the length of its arc.

$$\text{area of sector} = \frac{1}{2} \times r \times \ell$$
$$= \frac{1}{2} \times r \times \frac{m}{180}(\pi r)$$
$$= \frac{m}{360}(\pi r^2)$$

Definition: A segment of a circle is a region bounded by a chord.



To find the area of a segment, first find the area of the sector and then subtract the area of the triangle contained in the sector.

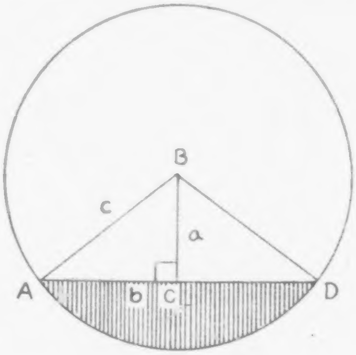
Finding the area of the triangle may be a bit of a job, especially if the central angle associated with the segment does not have a measure of 60°, 90°, or 120°. In this case, you'll be able to make use of the 30-60-90 triangle, the isosceles right triangle, or the equilateral triangle relationships. In other cases, you'll need to use numerical trigonometry. An example of how such a problem would be set up is given in the accompanying diagram:

$\overline{AB}$ is the radius. Draw BC $\perp$ AD.

Then $\angle$ ABC = $\frac{1}{2}$ the central angle.

sin B = $\frac{b}{c}$; cos B = $\frac{a}{c}$

Then, use the formula for the area of a triangle.



8. Find the lengths of these arcs.

a. r = 10, m = 40, ℓ = ______ c. r = 8, m = 90, ℓ = ______

b. r = 25, m = 42, ℓ = ______ d. r = 500, m = 150, ℓ = ______

9. Find the area of the sector of a circle with radius 8 whose arc has the following measure.

a. 60 ______ c. 90 ______

b. 150 ______ d. 160 ______

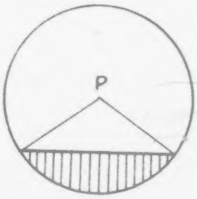
10. Find the area of the shaded region where given:

a. m $\angle$ P = 90, r = 12, ______

b. m $\angle$ P = 60, r = 4, ______

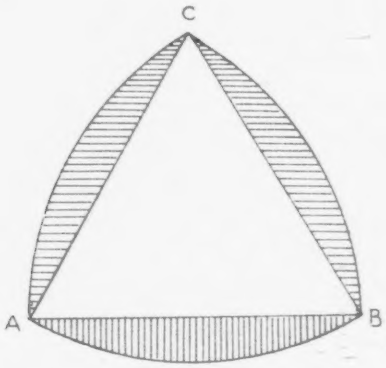
c. m $\angle$ P = 100, r = 6, ______

d. m $\angle$ P = 120, r = 2, ______



For a little variation, try the following:

11. Triangle ABC is equilateral and the radius of each arc is 12, find the area of the shaded region.



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II-1 Printing and Graphic Arts

PRINTING AND GRAPHIC ARTS

All of us have looked through the telephone book; few of us, however, have stopped to think of all the work, done by many different kinds of people, that has gone into these books. Even after the copy (material that goes on the pages) is ready, it takes much effort by many people to get the book prepared. Most of this work is done by people from the printing industry.

There may be as many as 25 different specialized jobs in this general industry, each with its own requirements, pay, opportunities, etc. Generally, however, the workers can be divided into three areas: printing craftsmen, preparation and management workers, and the allied skilled workers. The machine operators, pressmen, cameramen, etc., make up the craftsmen category. People ranging from reporters to accountants and executives make up the next category, and all of the other workers who help in the actual production of the material would fit into the area of allied skilled workers.

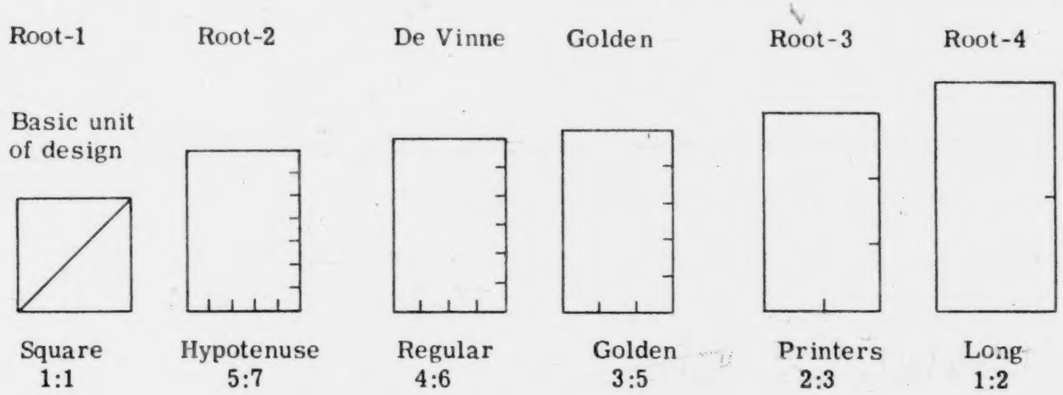
Most workers become skilled by going through an apprenticeship program. It may be very formal as is the case with most typesetters or it may be an informal program of on-the-job training as in the case for an artist or reporter. Another means for obtaining training would be to attend a vocational trade school. Many courses and programs are available and in some (the area vocational schools) there is no tuition if you are under 21 and a high school graduate.

The printing industry (graphic arts) is involved in the preparation of any kind of written material. The telephone book mentioned earlier, the textbooks we use, the newspaper, etc., are all prepared by some phase of this industry. Many of the workers become involved in math very quickly. For instance, in working with the printing of books, the type page (that is, the part of the page the printing covers) which will appear to be the most pleasing to the eye should be equal to one half the area of the paper page. Another way of stating this rule is that the printed part of the page should equal the area of the white marginal space surrounding it and the sides of the type page are to be proportional to the sides of the paper page.

Most book printing is in the form of a rectangle. Those rectangles most commonly used are:

Those rectangles most commonly used are:

Rectangles	Root-2	De Vinne	Golden	Root-3	Root-4
Factor	1.40	1.50	1.60	1.70	2.00
Reciprocal	.71	.75	.78	.82	.87
Type area (%)	50	55	62	67	75
Margin area (%)	50	45	38	33	25

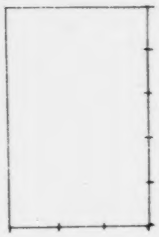


According to the table, which rectangle (notice the root-2 rectangle is also called a hypotenuse rectangle, etc.) most closely meets the rule on what is the most pleasing ratio of type area to margin area? 1.____________________

PRINTING AND GRAPHIC ARTS

If all you are interested in is getting the most type area on the paper, which oblong would you choose? 2.________________________________. Measure your geometry book (length and width) and determine which rectangle the publisher used: 3.________________________. If a book measures 10" by 6", which rectangle is used? 4.________________________. One measuring 15" by 21" follows the 5.______________________oblong.

The following is an example of a scale layout..



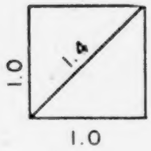
This would be a layout for a golden rectangle.

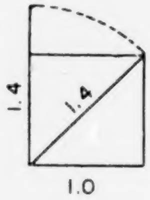
Using the same scale, make a layout for each of the following paper page dimensions and tell what type of printer's rectangle they form.

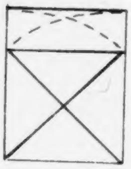
6. A paper page 9" by 6"

7. A paper page $2\frac{1}{2}$ by $3\frac{1}{2}$.

The rectangle used most often is the root-2 or hypotenuse rectangle. We can construct a root-2 rectangle by starting with a square. Follow the directions to construct a root-2:







A. Construct a square .

B. Draw the diagonal.

C. Raise the diagonal to a vertical position -- this gives you the height of the root-2 rectangle.

D. Finish the rectangle using the height dimension.

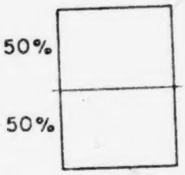
8. Start with a three-inch square and construct a root-2 rectangle. Begin the square low enough to allow for the diagonal.

To help determine where the typed material should be on the page, printers have three centers to judge with.

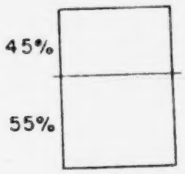
The math center = $\frac{1}{2}$ of page length

The optical center = .45 of page length measured from top of page

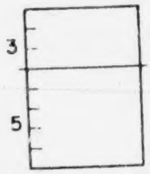
The line of golden proportion = $\frac{3}{8}$ of page length measured from top of page



MATH CENTER



OPTICAL CENTER



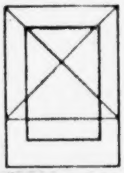
LINE OF GOLDEN PROPORTION

Notice the optical center (the areas of the page our eyes center on) is different from the math center.

9. Find the optical center of the root-2 rectangle in problem eight. (Use the diagram.) Answer:______________________________

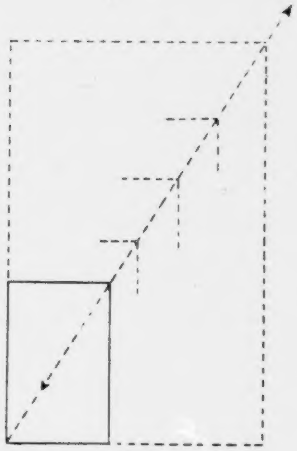
The composer may take the optical center into consideration and leave more margin on the bottom and extend the type to the top. Is this true with any of your school books?

Example



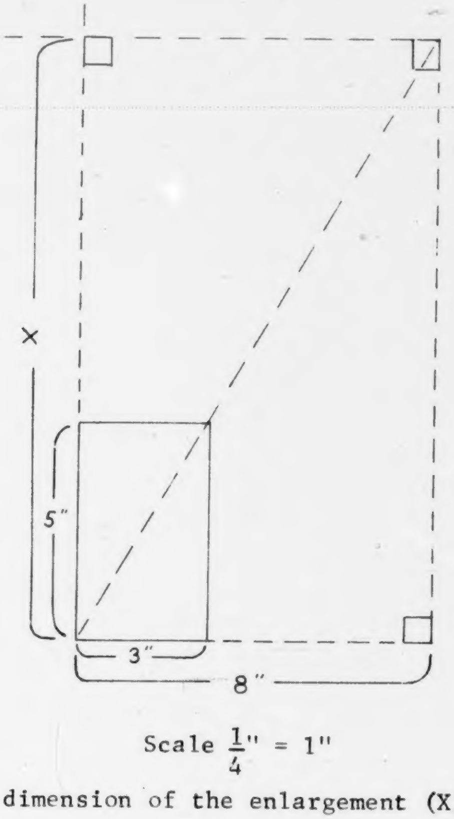
SINGLE PAGE
2:3 RATIO

When a composer wishes to enlarge or reduce a drawing, photograph, etc., he will often use the diagonal line method. The principle that rectangular areas enlarge and reduce proportionally along their diagonals is used.



DIAGONAL LINE METHOD

One dimension of the enlargement of reduction must be known. For example, a 3" x 5" (width x depth) picture is to be enlarged to an 8"-wide copy.



Scale $\frac{1}{4}$" = 1"

To find the remaining dimension of the enlargement (X) we would use the proportion formula on page 7.

FORMULA: $\frac{\text{width of original}}{\text{length of original}} = \frac{\text{desired width}}{\text{desired length}}$

thus $\frac{3''}{5''} = \frac{8''}{x}$

$3x = 40$ (cross multiplication)

$x = \frac{40}{3}$ or $13\frac{1}{3}''$

10. Find the remaining dimension of an enlargement of a 4" x 7" picture enlarged to a width of 11". Make a diagram similar to the example above with a scale of $\frac{1}{4}'' = 1''$.

The previous material gives a sampling of the type of geometry used in graphic arts careers. There are other math applications used in this trade that are not covered in this unit. Please feel free to ask your teacher or counselor for more information.

II-2 Heavy Equipment Operator

HEAVY EQUIPMENT OPERATORS

Heavy equipment operators work with many different kinds of construction equipment such as cranes, graders, bulldozers, etc. Over 310,000 of these operators are employed throughout the country and earn between $5.25 and $7.04 an hour. Most of these operators have completed an apprenticeship program consisting of up to three years of on-the-job training with some (144 hours) related classroom instruction. This related instruction is usuallly in areas such as equipment maintenance, electricity, welding, reading plans, etc.

Other operators attend school full time at a local trade school or receive the training while in the armed forces. In Minnesota, the Staples Area Vocational School provides a program in heavy equipment operation and maintenance that takes twenty-one months.

The government predicts that the number of equipment operators needed in the near future will increase rapidly. As construction activity increases and as highway construction continues, job opportunities will also increase. Larger and more efficient equipment will tend, however, to offset some of these increases.

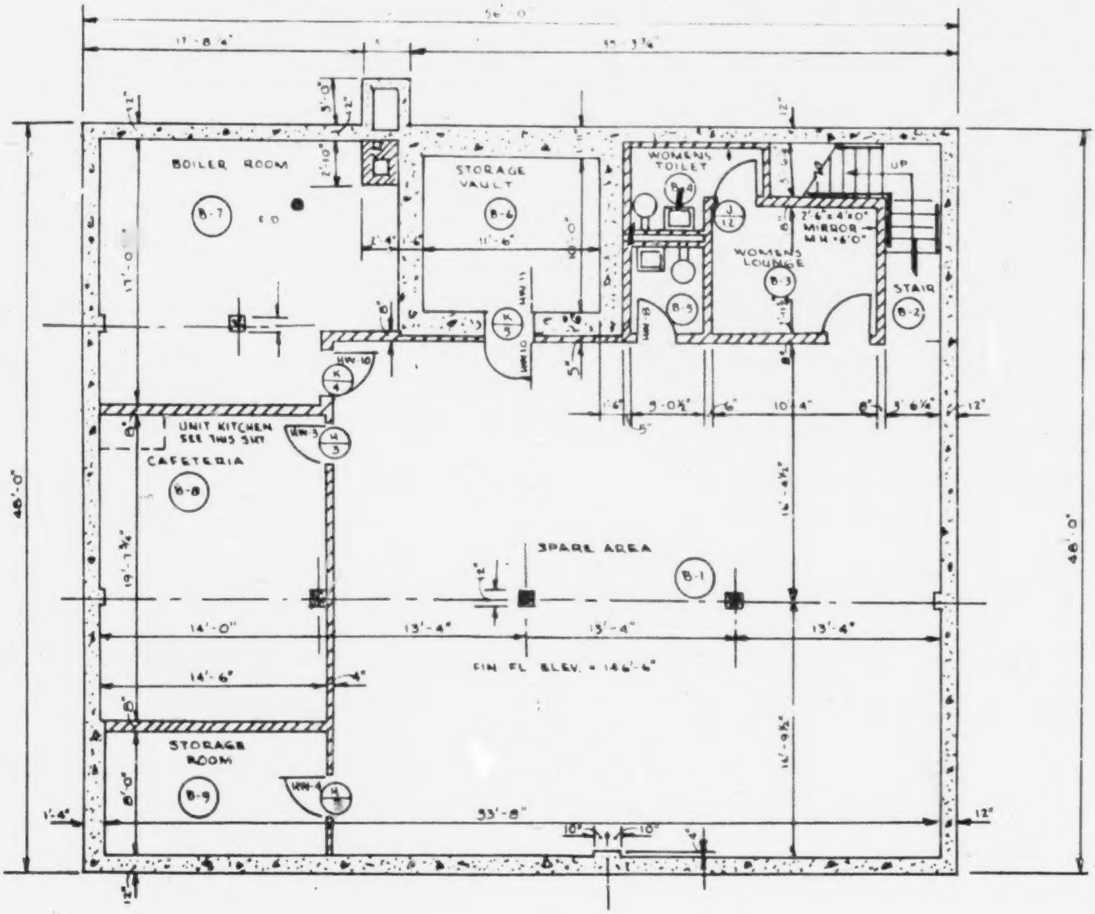
Operators need good hand-eye coordination and judgment plus mechanical aptitude and an interest and desire to work physically in the out-doors.

The heavy equipment operator must also have a complete understanding of blueprints as well as the ability to visualize the structure to be built.
He must also know the nature of different types of soils.

HEAVY EQUIPMENT OPERATORS

An example of a diagram of a typical job is the basement plan for a bank below.

Fig. 1



B A S E M E N T   P L A N
SCALE 1/8" = 1'-0"

The walls of the basement are to be block, so footings for the blocks must be parallel. The equipment operator must dig a trench for these footings. They are 12" wide and 10" deep and will go around the entire building; the walls of the basement will go 8' into the ground. In order to "price" this job, the total volume of dirt to be removed must be found. The estimator, working with the equipment operator, had to find the following information.

1. Total area of basement = ______________________________

2. Total volume of basement = ______________________________

3. Length of footings = (Total) ________ ________________

4. Volume of footings = length x width x height = ________________

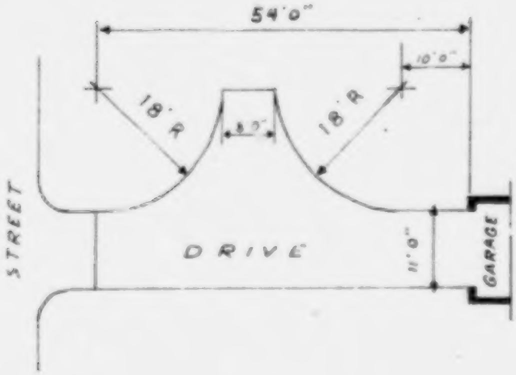
5. Total volume of basement and footings (Give answer in cubic yards.) =

______________________________ HINT: 27 cubic feet = 1 cubic yard

With this information, the correct number of trucks can be arranged to carry away the dirt. An accurate estimate of the cost can also be completed.

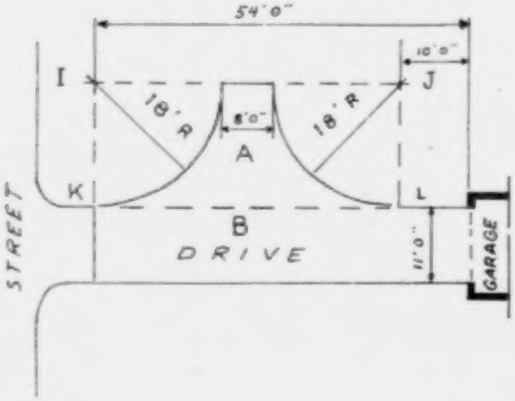
Many people who are making improvements around their homes act as their own general contractor. These people oftentimes rely on the knowledge of drivers for trucking companies to help them make decisions. For example, Tom Jones had a crushed rock driveway and wanted to concrete it. (See fig. 2.)

Fig. 2



Tom called the G & P dirt removal company to remove the crushed rock and go as deep as necessary for the concrete to be poured. They informed Tom that they charge $45 per hour for the use of their machines and labor, with a minimum charge of $50 for any one job. G & P also said that they can remove roughly 10 cubic yards of dirt per hour. Tom's next job as his own general contractor was to find the number of cubic feet in his driveway. He wanted to go 6" deep to allow for 2" of sand and 4" of concrete.

He drew a diagram of his Y-turn driveway similar to figure 2, added measurements, and divided it into two areas -- which resulted in figure 3. The area of B (see figure 3) would be that of a rectangle, so Tom had no trouble finding its volume. The area of A, however, was oddshaped. Tom decided to block off a rectangular area (IJKL), find the area of it, then subtract the area of the two partial circles (notice each is one-fourth circle) called sectors of a circle.



HEAVY EQUIPMENT OPERATORS

See if you can find the following needed information.

6. Area of B______________________________
7. Area of IJKL__________ ___________
8. Area of one of the sectors_________________________
9. Area of A (answer to 7 minus the answer to 8 doubled)_______________
10. Total area of driveway______________
11. Total volume of driveway (6" deep) in cubic feet ___________________
12. Total volume in cubic yards_________

With this information, Tom knew how much it would cost to remove the crushed rocks and dirt. (Remember the charge is $45 per hour and that they can remove 10 cubic yards per hour.) The cost would be 13.____________________________

HEAVY EQUIPMENT OPERATORS

The next day G & P sent out a couple of men and a backhoe-loader combination. The backhoe would dig out the old rock and dirt and the loader (on the other end of the backhoe) would scoop up the loosened debris and put it in the truck. The operator said the backhoe itself could hold up to one-half cubic yard of material, but because of the hardness of the soil, it usually held one-fourth cubic yard. The loader's scoop had a capacity of one cubic yard. The operator suggested to Tom that he order the sand as soon as possible. Already knowing the area of the driveway and that 2" of sand is needed, find the number of cubic yards of sand needed. (Hint: determine the volume of sand needed by multiplying area x depth in common units. Then change to cubic yards.)

14. Cubic yards of sand needed = ____________________

This same operator has handled some of the big cats, graders, and earth movers seen so often on highway projects. He said that heavy equipment operators must work their way up from small to large projects in steps. Experience is the key to success!

II-3 Fashion and Apparel Design

FASHION AND APPAREL DESIGN

Fashion and apparel design is an occupation that lends itself readily to a person with an artistic ability combined with creativity and an adventuresome spirit. Designers set trends in clothing styles and sell those "looks of the future" to the public. It is a demanding career, for if one works merely upon what is currently in vogue, then consumers will soon realize this and begin buying from one's competitors.

A fashion designer creates original designs for new types and styles of apparel. The creation of the new styles occurs in steps; first, a sketch is made, then approval is given by management, and finally an experimental garment is made. The designer is almost always directly involved in all of these steps. Therefore, an artistic ability, and ability to "sell" your new idea, and an ability to actually make the product are all important for the designer to have.

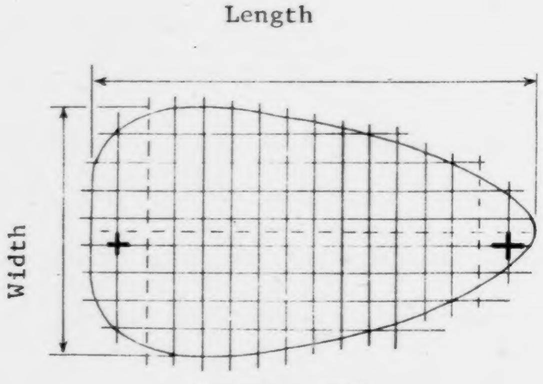
As is true for most jobs involving creativity, the designer will probably have much freedom. She/he may travel in order to pick up new ideas and much time and effort is consumed before definite styles are developed. The pressure to produce is present however, even with this freedom, because it is easy to judge how well a designer is doing by the number of successful new styles developed over a period of time. Style must be developed several seasons and sometimes a full year ahead of the production of the garment, so ideas must be several steps ahead of current fashion.

Much like other creative fields, the prospective designer enjoys the possibility of great reward -- both in fame and in fortune. The number of designers who "make it" to this point is very small compared to the many who aspire. Many would-be designers end up in related fields because the competition is so great in the area of design. These related fields include wholesale buying for stores, copying of successful designs, sales or advertising, or maybe even working in the actual production of a garment.

Most of the basic ideas covered in this unit would be covered in much more depth in any sewing course. When this is studied in a home economics course, one doesn't realize the large amount of mathematics which is involved.

For example, two of the basic pieces of equipment which will be used by anyone seriously interested in tailoring are a pressing cushion and a pressing edge. The pressing cushion is needed to help press darts in fabric pieces of the blouse and skirt to create curved lines which will drape more attractively over the body, whereas the pressing edge is used to press open new seams along their line of stitching without disturbing the remainder of the fabric. Using the scale given below, what is the length 1.__________ and width 2.__________ of the pressing cushion?

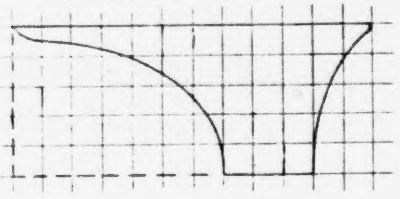
The pressing cushion can be made and filled to a solid pack with wool batting or sifted sawdust.



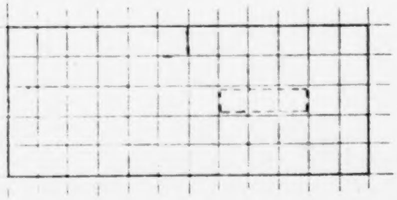
(1 square = 1 inch)

3. Using the scale given for the pressing board, draw the board to the proper size.

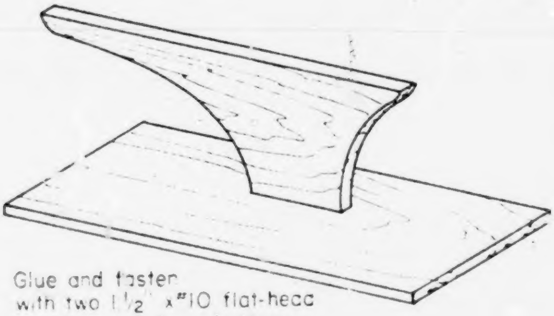
(1 square = 1 inch)



PRESSING EDGE 1" Board 5" x 12"



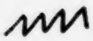
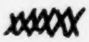
BASE 1" Board 5" x 12"



Glue and fasten
with two 1 1/2" x #10 flat-head
wood screws from bottom

The pressing board can be handmade by using a one-inch board, 5" by 12", and cutting a long tapered point on one end as shown above. The narrow edge is fastened upright (with two screws) to a second board, 12" long, to form a stand. The end with the tapered point is used for pressing.

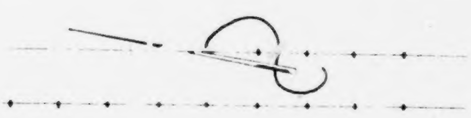
As you can see, most diagrams involve proportions, such as one square = one inch as used here. Later, you'll have the chance to do more "blow-ups" of simple patterns.

An area of geometry not studied too much at first is called translations. It basically means having a pattern of some sort duplicated again and again along a straight line, such as  ,  , etc. A type of transormation results in a series of parallel lines , which may be thought of as lines which are "translated" along a vertical line.

FASHION AND APPAREL DESIGN

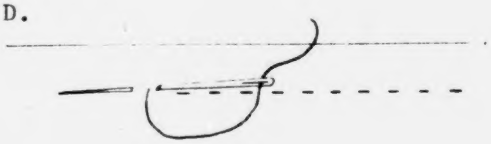
You're probably asking just what all this has to do with tailoring. Frankly, a lot, for this type of geometry is applied in the most fundamental ingredient of good tailoring, the stitching. All stitches have a purpose, and, if you examine good stitching closely, you'll find that it is all composed of repetitive patterns, or translations. Notice the translations in the following common stitches.

A.



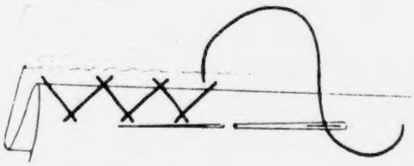
Blind Hemming

D.



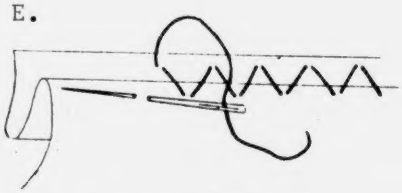
Backstitch

B.



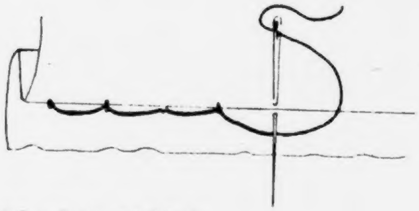
Catch stitch

E.



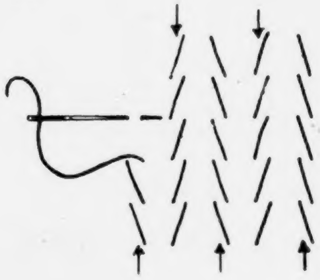
Floating or Slipstitch

C.



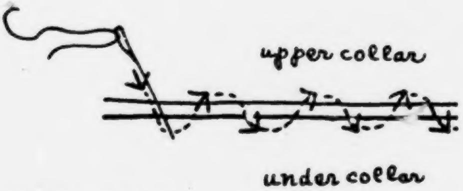
Blanket Stitch

F.



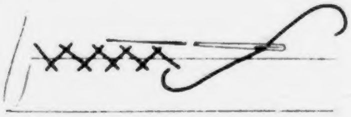
Padding stitch

G.



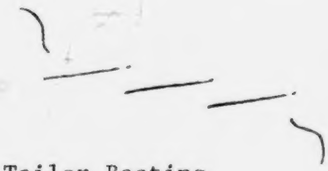
Stab Stitch

I.



Catchstitch

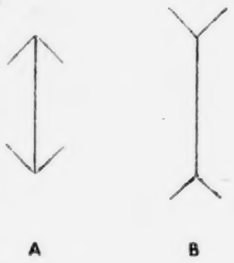
H.



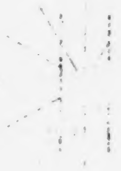
Tailor Basting

Can your eyes always believe what they see? Through the clever use of optical illusion, it is possible to deceive the eye. There are many examples of this fool-the-eye magic. Here are some of the more familiar.

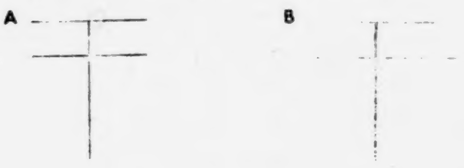
Which line is longer? Of course, the lines are the same length, but the optical illusion makes you think otherwise. Deflecting the gaze downward in A seems to shorten the line. Extending the line as in B makes the line seem longer.

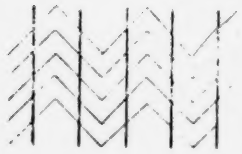


Do the vertical lines bulge in the center? Although it would appear so, this is merely an optical illusion. The lines are parallel, but the many lines that converge upon one point call attention to that area, making it appear wider.



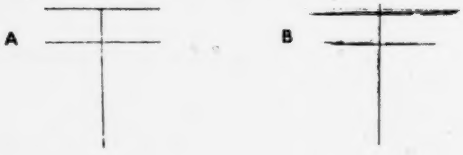
Is A longer than B? No, but the longer line under B makes B look shorter.





Are the vertical lines parallel? Yes. The zig-zag of the opposing lines only makes them appear to waver.

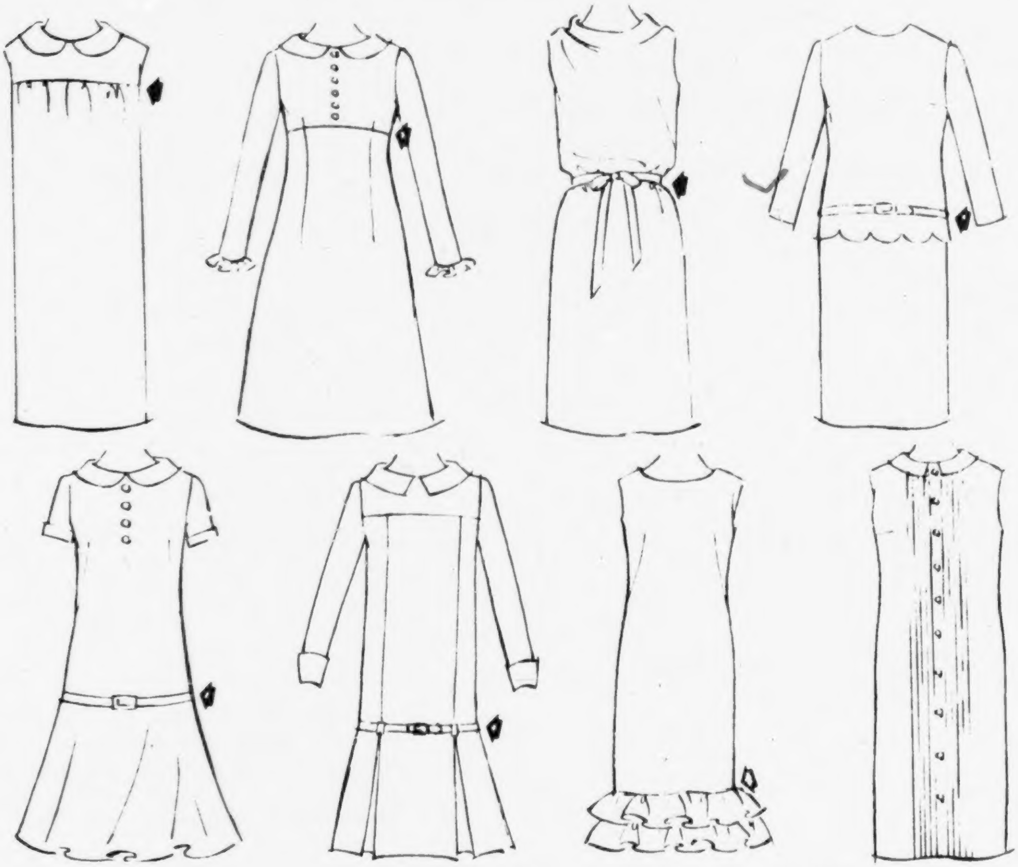
Which line is longer, A or B. Both are the same length, but A seems to be longer because the line above it is the same length. A longer line or added width at the top, diminishes the size of the lower line.



4. Find optical illusions in clothing at home. Draw examples of clothing that will make the figure appear taller, shorter, heavier, and more slender. (If you can't draw well, bring at least 10 pictures).

5. Find examples of clothing using dominant vertical or horizontal lines that carry the eye across or up and down the design. (Find or draw at least 10 pictures.)

Good proportion in spacing and a pleasing relationship of sizes and shapes result in a design that is harmoniously related and unified.

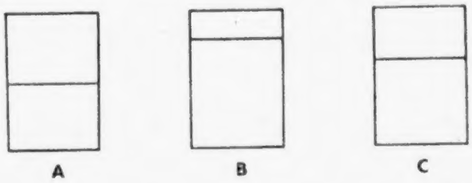


Simplicity Pattern Co., Inc.

The location of the waistline divides the silhouette and creates a proportion relationship.

FASHION AND APPAREL DESIGN

The principle of proportion is concerned chiefly with spaces, sizes, and shapes within a design and their relationship to each other. To illustrate this principle, examine the rectangles. In both A and B the division of space is very obvious and therefore considered not as satisfying as C.



In general, an uneven yet similar relationship is more interesting than either an even or completely dissimilar one. To be pleasing and in good proportion, the relationship should be neither too obvious nor too hard to see.

There are some laws of proportion in geometry that mathematically describe the best possible relationship between areas. In the rectangles just shown, we saw that the 1:1 ratio (with parts the same size) in A and the 1:2 ratio (with the larger part twice as large as the smaller) in B are not pleasing because they are too obvious. The pleasing ratio of rectangle C approximates the 1 to 1.618 ratio. This is known as the "golden ratio."

This golden ratio, 1 to 6.___________, enters the picture in designing width and placement of borders, width of bands and their spacings, placement of crosswise seams, size of pockets, and spacing of buttons.

7. Find a picture of some clothing that illustrates the golden ratio design. Draw the rectangles on the picture to show this.

8. Experiment with the placement of the waistline seam and the length of a jacket on identical silhouettes. What effects do the placement of the waistline and jacket length have on the figure?

9. Analyze several garments and classify them as good and poor design. (Get pictures from magazines.) Give reasons for your decision.

This completes the few examples of geometry principles as applied in fashion tailoring. However, geometry is just a start. One really interested in this field needs good training in both sewing and art, for both of these comprise the backbone of this career.

II-4 Navigation

NAVIGATION

Navigation had always fascinated Dan. Maybe it was more the dream of sailing the high seas or flying to far-away places. But for whatever reason we make choices for our life's occupation, Dan wished to investigate navigation as a career.

Dan's search for information took him to the school's counseling office where he discovered that much of today's navigation is done by instruments with help from computers. He learned, however, that all of the people responsible for getting a ship or an airplane from one place to another must be able to figure navigation routes "manually." In air transportation, most navigation is done by the copilot and he probably received his training in a very specialized school or in the armed forces.

Special licenses are required for most people involved in public transportation. Tests are taken in order to show that one can properly do what is required. New equipment means further training and the navigator, as well as pilot and copilot, find advancement and pay increases often tied to their ability to do the complete job well.

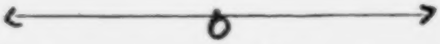
The great responsibility a navigator has necessitates that he does his job well. This responsibility also allows him to command much as far as pay, working conditions, and fringe benefits are concerned. Navigators enjoy most of the benefits as copilots do. These include up to $30,000 a year in salary, a flight schedule of 60 to 85 hours per month, and generally the finest working conditions. The burden of responsibility is great, however, and the need to keep alert and well prepared at all times is required. During his investigation,

NAVIGATION

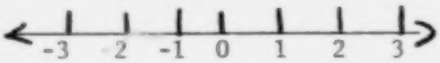
Dan found that in the early days of navigation a need for distinguishing one point on the earth from another was needed. The earth, being two-dimensional (height and width) required a different system of locating a point from determining a one-dimensional line. With the line, all that is necessary is to establish any one point, give it a number (usually zero), choose how long one unit will be, and mark of all other points.

Example:

a. Draw a line

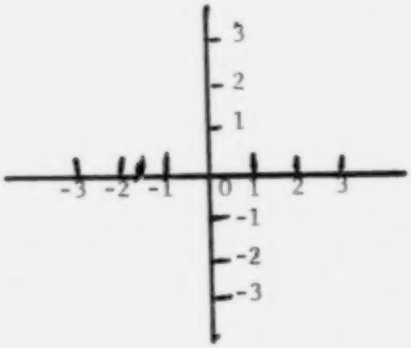
b. Establish a point 

and how long one unit is

c. Mark off all other points as follows 

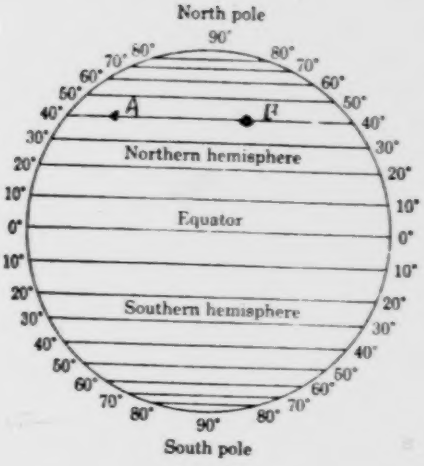
The "unit" of measure can be anything: feet, miles, degrees, etc.

The earth as a two-dimensional object will need two measuring lines, one for height, another for width. We can show the ideas as follows:



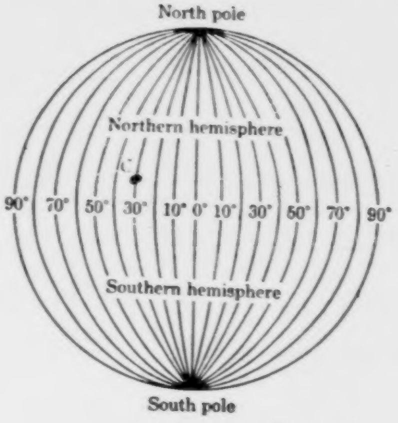
With this idea in mind, Dan was shown the application navigators use on the earth. The measurement of height is done by lines of latitude; they enable us to tell how many degrees north or south of the equator (the exact middle) a given place is.

Notice that each line in the diagram is parallel to the others, thus sometimes the lines of latitude are called parallels. To distinguish the 10° latitude above the equator from the 10° latitude below, latitudes above the equator are called north latitudes, and latitudes below are called south latitudes. Thus point B is at 40° north latitude. But so are an infinite number of other points (as point A).



Parallels

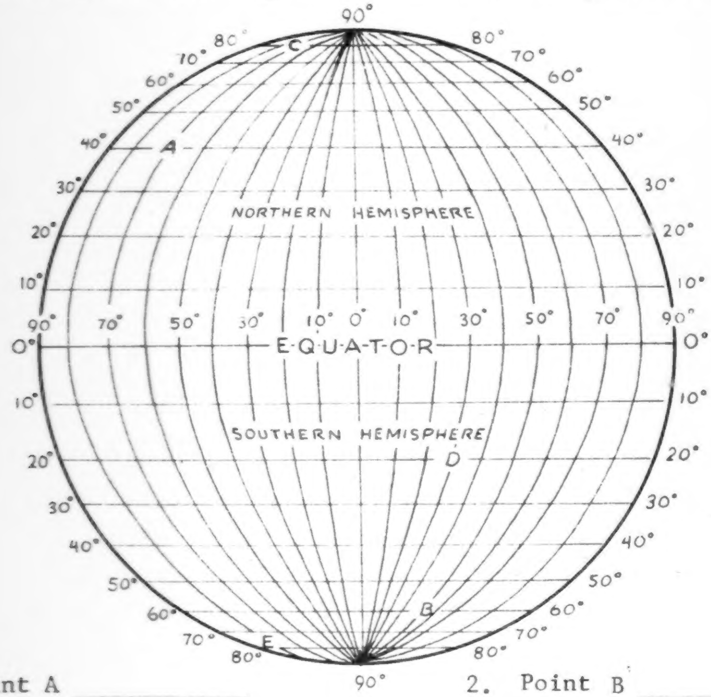
The second dimension, width, is measured by lines of longitude called meridians. Lines of longitude are imaginary lines running from north to south. Again notice that degree readings are symmetrical with respect to the 0° meridians. (0° meridian was chosen to go through Greenwich, England.) The degree meridians to the left of the 0° meridian is a west longitude reading, those to the right are east longitude reading; thus point C is 30° W.



Meridians

NAVIGATION

Find the latitude and longitude of each of the points on the graph.

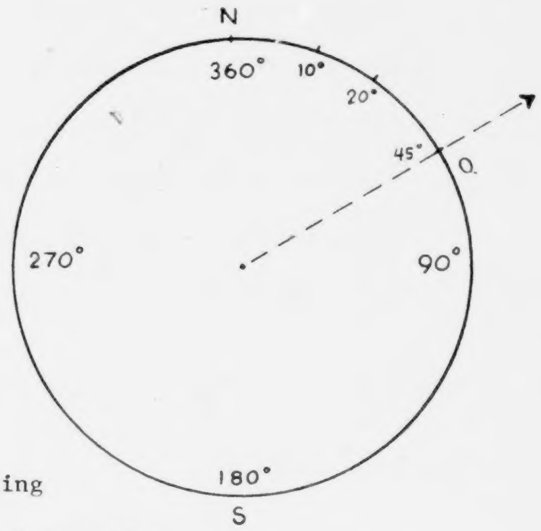


1. Point A ____________
2. Point B ____________________
3. Point C ____________
4. Point D ____________________
5. Point E ____________
6. Analyze the diagram and find two points that appear to have more than one longitude running to them . ______________________

We have stated that meridians or lines of longitude are drawn from the North Pole to the South Pole. In order to guide the air navigator, a mariner's (or magnetic) compass has been used. All directions on the compass are measured from true north.

NAVIGATION

Mariner's compass by 10 mark's



The 0° mark must be pointing to true north. Then all that must be done is to align the direction of the airplane through the center of the compass to whatever direction it is going. If the direction is through point Q, a reading of 45° T("T" stands for true north) would be taken.

On the compass, plot the following readings by starting at the center and drawing a dotted arrow through a point on the circle. Label the points on the circle by problem number.

7. Due south
8. 270° T
9. Northwest
10. 110° T

Use the compass as reference to answer the following questions.

11. Find the reading for a course of due east. ____________________
12. Find the reading for a course running southeast. ______________________

Don developed a new appreciation for circular measuring after working with the mariner's compass. One question that stuck in his mind was about the affect wind had on the airplane's course. It seemed reasonable that with no wind all a navigator would have to do is set a course at the correct angle from the point of departure to the destination. He was soon told that the most difficult part of the navigator's job is to make heading corrections caused by wind variables.

NAVIGATION

To see the effect that wind has on a plane, Dan was given some graph paper and the following information:

Direction of the plane is 90° T.

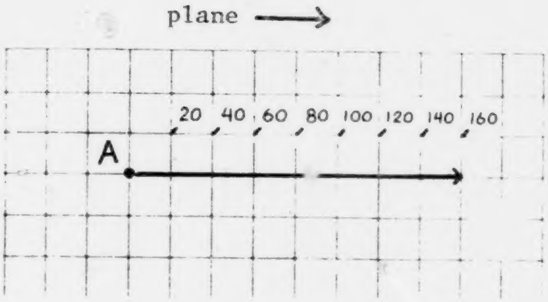
Velocity of the plane is 160 mph.

Direction of the wind is 180° T.

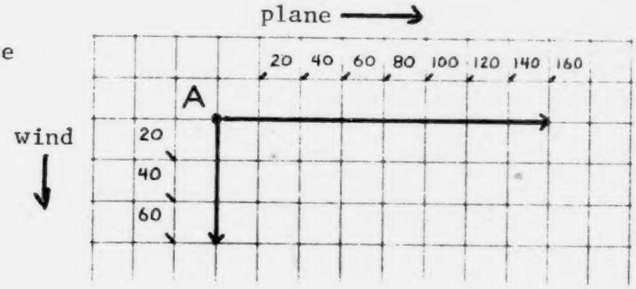
Velocity of the wind is 60 mph.

Dan's investigation contained the following steps.

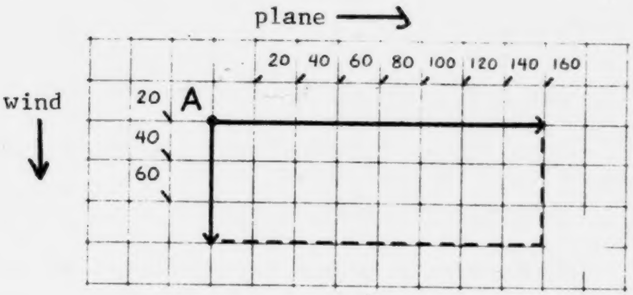
Step One: Starting at point A, Dan plotted the original direction and velocity (using one square = 20 mph).



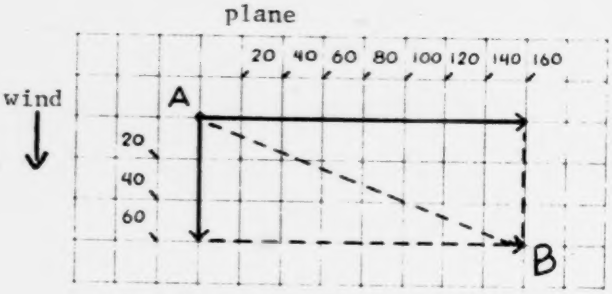
Step Two: On the same diagram, he plotted the wind's direction and velocity.



Step Three: He then formed a rectangle by drawing in the dotted lines.



Step Four: Dan could now find the resultant ground speed and direction of the plane by drawing the diagonal of the rectangle from point A.



resultant direction and velocity of plane

An easy but often inaccurate method of finding the resultant speed is to mark off the length of the diagonal on paper and then measure it by using the graph paper.

The resultant direction is formed by measuring the angle formed by the diagonal. Use a protractor to find this. (Ask your teacher for help if you do not have or know how to use a protractor.)

How far off course has the wind blown the plane?

13. ______________________

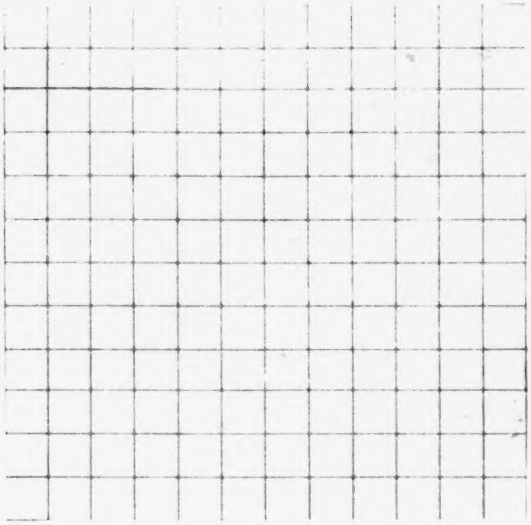
14. The new direction of the plane, if not corrected for wind, would be

a.________(Be sure to figure in the 90° T heading the plane started on.)

b. The resultant ground speed (AB) = ____

NAVIGATION

An airplane is heading due east. Its air speed is 200 mph. There is a wind of 90 mph that has a direction of 180° T. Find the resultant speed and direction of the plane (have each unit represent 20 mph).



15. The resultant ground speed is ______________.

16. How far off couse has the wind blown the plane? ____________________

17. What is the resultant direction of the plane? ____________________

II-5 Painting and Paperhanging

PAINTING AND PAPERHANGING

Bob's summer job as a painter's helper in his small home town was fine but he really wanted to get into a larger community. He decided to check into becoming a painter in some larger places. He wrote to the International Brotherhood of Painters and Allied Trades, 1925 K St. NW, Washington, D.C. 20006. The information he got back plus some checking with an area apprenticeship council gave him some answers.

He found that his apprenticeship and related training program would be much the same as the other building trades. He discovered that the average hourly pay for the building trades was $5.95. This didn't bother him because the pay varied much from city to city. In fact, he found this table in the Occupational Outlook Handbook.

	Rate Per Hour	
City	Painters	Paperhangers
Atlanta	$ 5.95	$ 6.20
Boston	6.08	
Chicago	6.35	6.35
Cincinnati	6.23	6.85
Detroit	7.00	7.00
Houston	5.34	5.44
Newark	6.00	
New Orleans	4.38	4.38
Philadelphia	5.22	5.34
Salt Lake City	4.87	5.07
San Diego	6.49	6.99
Spokane	6.17	6.17

PAINTING AND PAPERHANGING

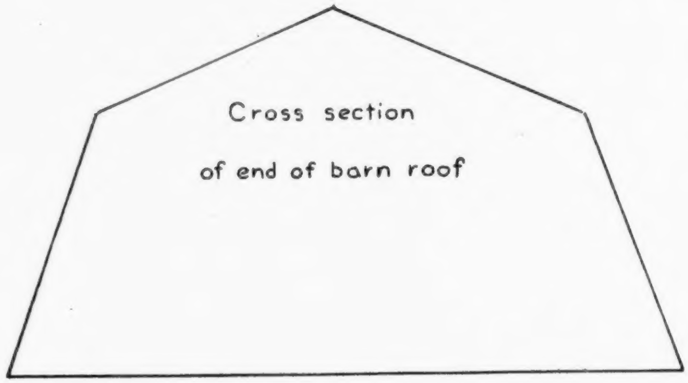
After further investigation, Bob found that being a painter or paperhanger involves more than just applying paint and paper. He would have to be able to mix paint, match colors, and have a knowledge of color harmony.

Many painters and paperhangers work for contractors involved in new building construction. Many others are also employed by contractors to do repair and redecorating work.

Painters and paperhangers may become foremen or advance to jobs as estimators for painting and decorating contractors. In this capacity, they would need to know how to compute material requirements and labor costs.

Some painters and paperhangers may establish their own businesses. This is what Bob thought he would eventually like to have -- his own painting and paper hanging business. The mathematics required was mostly involving area and volume. Here are some examples of problems one might encounter in this trade.

1. The cross section of the roof of a barn has the shape of an isosceles trapezoid surmounted by an isosceles triangle. The bases of the trapezoid are 50 feet and 34 feet and its altitude is 14 feet. The altitude of the triangle is 7 feet. Find the area of the cross section. Area = ______________.



PAINTING AND PAPERHANGING

2. One gallon of paint covers 400 square feet on weather boarding. Find the number of gallons necessary to paint the two ends of the barn in problem 1.

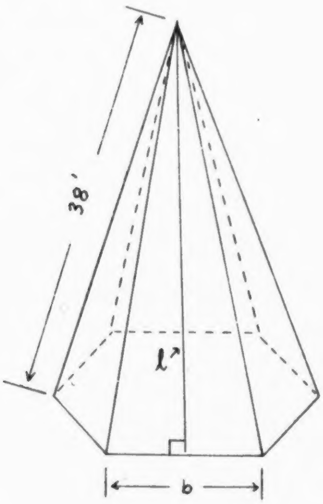
   Gallons = ____________________

3. A church steeple has the shape of a regular hexagonal pyramid. A lateral edge of the pyramid is 38 feet and each side of the base is $7\frac{1}{2}$ feet. The average covering capacity of buff paint on weather boarding is 400 square feet per gallon, one coat. Find the number of square feet of the steeple and the number of gallons necessary to give the steeple one coat of paint.

   a. Square feet = ____________________

   b. Gallons = ____________________

   Hint

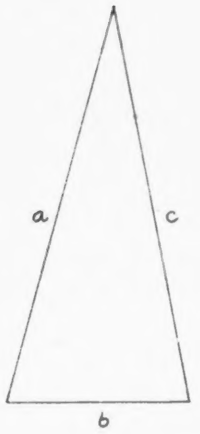


ℓ = lateral height (altitude of triangular face)

There are six congruent triangles. A = $\frac{1}{2}$.e.b

Find ℓ by Pythagorean theorem:

$(\frac{1}{2}b)^2 + \ell^2 = 38^2$



PAINTING AND PAPERHANGING

Another way to find the area of each triangle is to use the formula $A = s(s-a)(s-b)(s-c)$ where $s = \frac{1}{2}(a+b+c)$ and a, b, and c are the three lengths of the sides of the triangle. With this formula it is not necessary to know the altitude of the triangle.

4. Find the number of gallons of paint needed to paint the sides and tops of three gasoline storage tanks, each 12 feet in diameter and 16 feet high. Apply three coats of paint. One gallon of paint covers 500 square feet with one coat.

a. Gallons = ____________________

At $5 per gallon, find the cost of the paint. b. Cost = ____________________

5. What will be the cost of painting the walls and ceiling of a room 12 feet 6 inches by 16 feet and 8 feet 4 inches high at 80 cents per square yard. Cost = __________

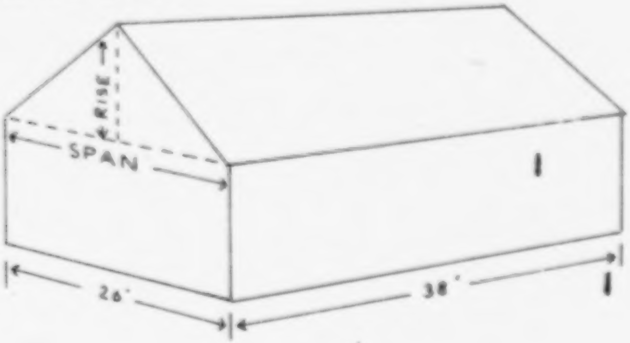
6. The four side walls of a room 14'-6" wide and 17'-8" long are to be wallpapered. The walls are 8 feet high. Allow 54 square feet deduction for doors and windows. How many square feet of surface will be wallpapered? a. Square feet = ________

Find out how may square feet one roll of wallpaper will cover. b. ____________

How many rolls are needed to do the job? c. ______________________________

Find out the cost of vinyl wallpaper and figure out how much it would cost for the wallpaper to cover these walls. d. Cost = ______________________________

7. Find the number of gallons of paint needed to paint the building pictured. The length of the building is 38 feet and the width is 26 feet. The height to the eaves is 18 feet. The roof is the gable type, the pitch being one fourth. The building is to be given two coats. One gallon covers 400 square feet with one coat.



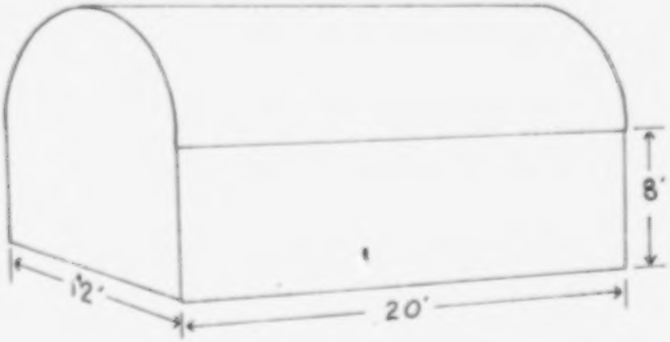
Pitch = $\frac{1}{4}$ means $\left(\frac{\text{rise}}{\text{span}} = \frac{1}{4} \text{ or rise} = \frac{\text{span}}{4}\right)$

You can figure out the height of the gable end from the pitch.

The height of the gable end is ______________________ feet.

8. A building having a semi-circular roof is 20 feet long, 12 feet wide, and 8 feet high to the eaves. Find the total area of the building (excluding the roof) to be painted. ($A = \pi \times R^2$. Use $\pi = 3.14$.)

Area = ______________________________



9. If three pints of thinner are used in 5 gallons of paint, how many gallons of thinner are needed in 48 gallons of paint? Gallons = ____________________

10. A utility cabinet is to be painted inside and outside. It has four shelf spaces (three shelves). If it is 50 inches by 24 inches by 12 inches, what is the total surface in square feet to be painted? Disregard the thickness of the material and shelves. (Shelves are 24" x 12".)

Area = ____________________

II-6 Landscape Technology

LANDSCAPE TECHNOLOGY

Jack thought that some day he would like to get involved in agriculture. He decided to find out what was available for careers besides farming He was more interested in land development and plants than in livestock.

After a little investigating, Jack found there were three very closely related jobs that interested him:

1. Landscape architect
2. Landscape gardener
3. Greenkeepers

These occupations involve landscaping and caring for grounds. The "landscape architect" plans and designs land areas for all sorts of projects such as parks, airports, highways, schools, and commercial and industrial sites. Jack found that if he were such an architect he would prepare site plans, working drawings, specifications, and cost estimates for land development. He could also supervise the construction work based on his plans.

The "landscape gardener," Jack discovered, plans and works on small-scale landscaping and maintaing the grounds of homes and institutions. He might prepare and grade the terrain, apply fertilizer, seed the lawn, or sod lawns. He locates where to plant shrubs, trees, flowers, and provides or supervises their care.

The "greenkeepers" supervise and set up details for workers to keep a golfcourse in good playing condition. Here Jack would determine work priorities and assign workers to specific tasks, such as fertilizing, seeding, mowing, raking, and spraying.

LANDSCAPE TECHNOLOGY

Jack was concerned with how much mathematics was needed to become a "landscape technician" of some type. He went to a local vocational school and got some sample problems. He also got himself a job at the Green Giant Garden Center, hoping to be able to get some practical experience.

At the area vocational school, Jack discovered that this course of study takes 18 months to complete. He found out that some of the topics covered are: plant materials, turf culture, landscape planting, sales, and management, and all students spend three months on the job with their work supervised by both the employer and the school. Jack was curious about what kind of work he would be doing both during the on-the-job part of the course and after. He was told that most technicians work for government agencies in the parks, golf courses, and orchards that they operate. Some work for private individuals or corporations doing the same kinds of things. A growing number work for nurseries or landscaping services.

It seemed to Jack that this was just what he would like to do and the job didn't require the college training that a landscape architect would have to take. It also would give him more of a chance to be outside, doing the actual work of preparing and managing the landscape.

Let's look at some typical problems Jack could be required to do.

1. You have been asked to fertilize the trapezoidal field pictured below by applying three pounds of nitrogen per 1,000 square feet. Suppose fertilizer costs $.45 a pound and the analysis is 24-16-12.

LANDSCAPE TECHNOLOGY

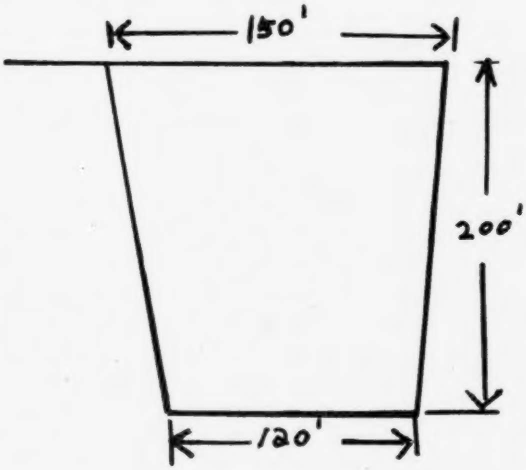
a. Find out what the analysis 24-16-12 means. ______________________________

b. How many square feet is the field?_____________________

c. What will the fertilizer cost? ____________________________________

$A = \frac{1}{2}h(a+b)$

A = area, h = height, a and b = lengths of the parallel sides.



2. The directions of a pesticide read three tablespoons per gallon of water. How many cups would you put in 50 gallons of water? cups = ______________________

Table

1 tablespoon = 3 teaspoons

1 ounce = 2 tablespoons

1 cup = 6 ounces

3. Use the picture on the next page.

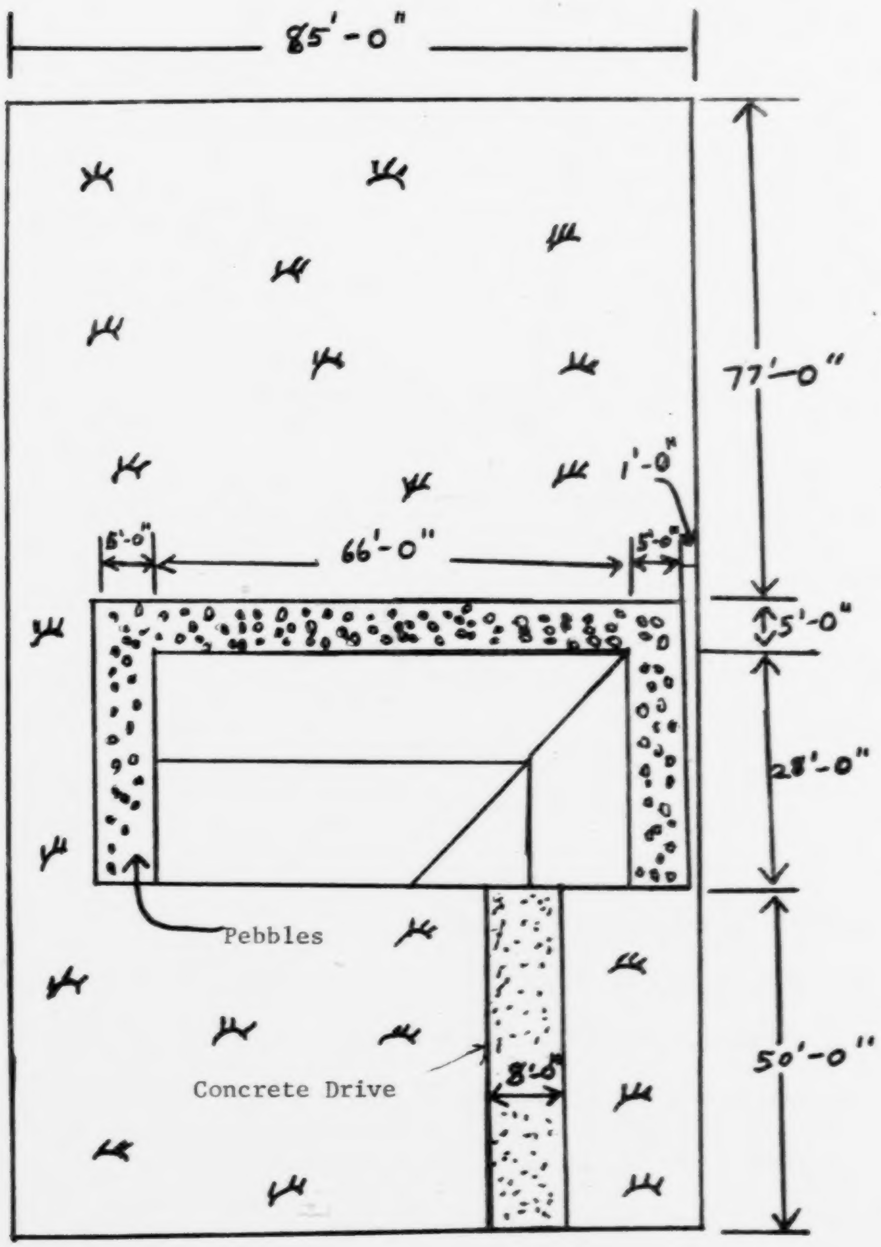
   a. How many tons of pebbles would be needed if it is spread 4" thick? (Figure 1 ton = 1 cubic yard) tons = ______________________________

   b. How many feet of redwood 2" x 4" would be needed to outline the pebble bed?_______________________

c. How many cubic yards of black dirt would be required if you spread it 4" thick on the lot where grass is required? ________________

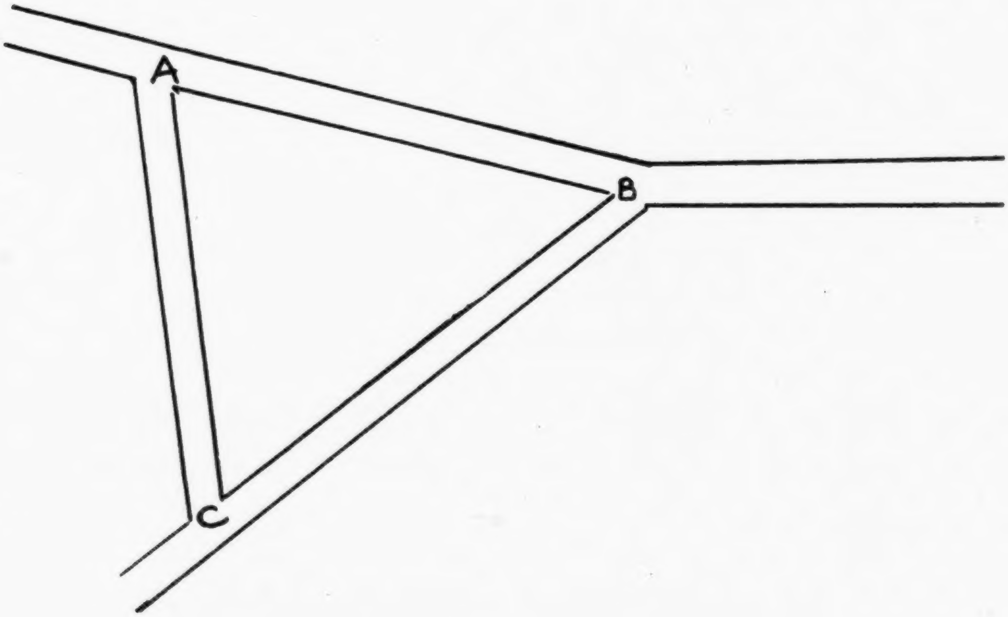
d. How many pounds of seed would be required if you apply one pound per 300 square feet?

House-yard Plan



LANDSCAPE TECHNOLOGY

4. a. Locate a tree so it will be equidistant from three roads: AB, BC, and CA shown below. (Use a compass and straight edge. Show your construction marks.)



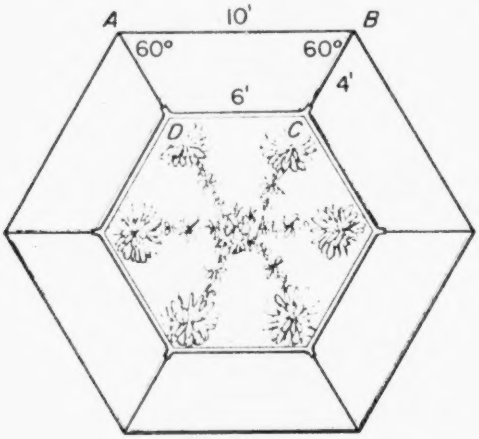
b. Locate three shrubs so that they are equidistant from the tree and each of the corners A, B, and C. (Use a compass and straight edge. Show your construction marks.)

5. Find the cost of sodding a lawn 132 feet wide and 152 feet long at $.56 per square yard.__________________________

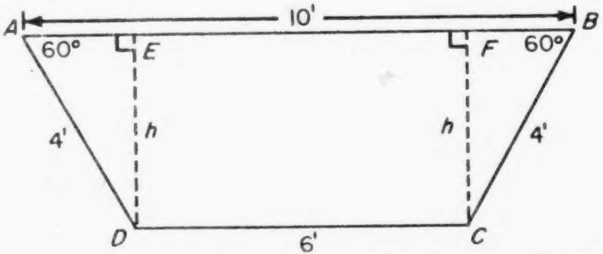
6. The formula for finding the area of an ellipse is $A = \pi \cdot a \cdot b$ where $a = \frac{1}{2}$ the major axis and b is $= \frac{1}{2}$ the minor axis. If you make a garden in the shape of an ellipse whose long diameter is 22 feet and whose short diameter is 18 feet, what is the area of the garden? Area = ____________________

7. Find the total area of a regular hexagonal walk surrounding a flower bed.

Area = ______________________________

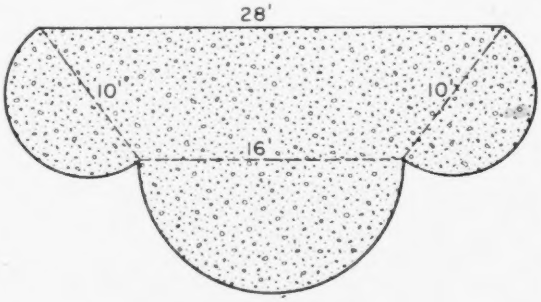


The walk is made up of six trapezoids. Since all six trapezoids have equal areas, solve for the area of just one and multiply by six. A representation of each trapezoid is shown here.



$A = \frac{1}{2}h(b_1 + b_2)$ (formula for the area of a trapezoid)

8. Find the volume of concrete needed for the patio pictured, if the depth is 4". (Hint: first find the area of the trapezoid, then the areas of the three semicircles.)



II-7 Carpenter

CARPENTER

Ever since Bill had watched some carpenters build a house next door, he thought it would be fun to build a house of his own some day. Maybe he would become a carpenter.

Bill knew that a neighbor had been a carpenter at one time and went to see him. While talking to the neighbor, Bill discovered that he was working at a local technical school, so could tell about the schools, too.

The neighbor told Bill that carpentry is the largest of all of the building trades, numbering over 830,000. He found that as the construction industry grows, so will the need for carpenters. When Bill's neighbor left carpentry, he was earning $6.60 an hour, but told Bill that wages run from $4.45 to $8.10 an hour depending upon the city. Most carpenters also receive some fringe benefits such as insurance and pension funds.

The apprenticeship program for carpenters usually lasts four years. The apprentice must also attend a twelve-hour per month related training class. During this time, the apprentice receives from 50 percent of journeyman's rate of pay to 85 to 90 percent at the end of the four years.

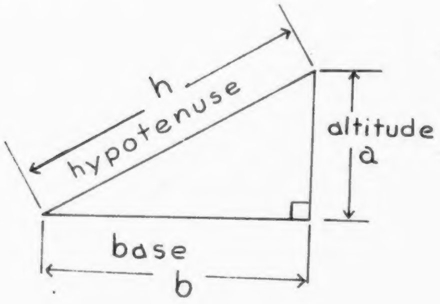
Some prospective carpenters would go to the school where Bill's neighbor was working. This school is a local area technical school (tuition free for high school graduates under 21 years of age). At a school like this, everything from blueprint reading to "stairways" would be studied. Other units would include framing, tool care, safety, construction problems, human relations, and communications. Courses at these schools usually run for one-and-one-half years. Some of the apprenticeship time could be spent in one of these trade schools.

CARPENTER

Bill was taking a course in geometry that year. He knew that his math courses in the past could help him if he wanted to do carpentry work because of the need to be able to compute with fractions and decimals, but how could the course of geometry be used? He decided to explore the possible uses of geometry in laying out and building a house.

While talking with the former carpenter now teaching at a local technical vocational school, Bill was given some examples of how he could apply his present course of geometry.

One of the first problems encountered is setting up the lines of excavation for the foundation of a building. Here he found that the formula called the "Pythagorean Theorem" was very useful. Bill knew that for any right triangle, $h^2 = a^2 + b^2$, where h represented the length of the longest side (hypotenuse) and a and b represented lengths of the shortest two sides (the altitude and base).



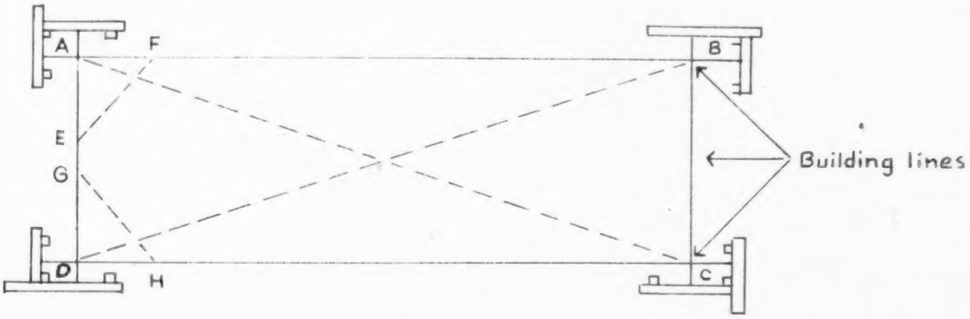
Now if $h^2 = a^2 + b^2$

then $h = \sqrt{a^2 + b^2}$

$b = \sqrt{h^2 - a^2}$

$a = \sqrt{h^2 - b^2}$

1. In the figure below, rectangle ABCD represents the lines of excavation for the foundation of a house. Figure the lengths of the diagonals $\overline{AC}$ and $\overline{BD}$ if the side AB is 49'-0" and AD is 36'-0".



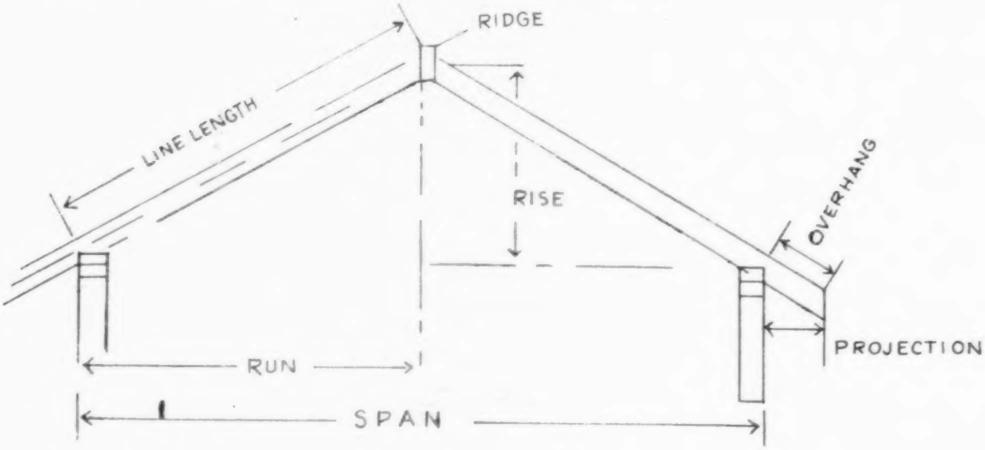
Diagonals are ________________ and ____________________.

2. In checking the corner D to find if it is square, GD is laid off equal to 3'-0" and DH is laid off equal to 4'-0". What should the length of the segment $\overline{GH}$ be? __________________

3. To square up corner A, we lay off AE equal to 6'-0" and AF equal to 8'-0". What should we then make the line EF? ________________________________

4. Rafters had to be ordered for the building. Bill was asked to figure out what length rafters he should purchase for the building. In the illustration on the next page what is the line length of the common rafter of the gable roof shown if it has a span of 36'-0" and a rise of 9'-0"?

4. Line length = ____________________



5. If you want an overhang of 1'-6", what length of lumber must be purchased to make common rafters for the building? Rafter = ____________________

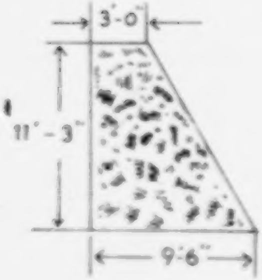
6. It was necessary to order sheeting for the walls and gables of the building. Using the dimensions of problem 4 and knowing the height of the walls to be 8'-6", find the area of one gable end of the building.

Area = ____________________

What is the least number of 4' x 8' sheets of plywood it would take to cover one gable end. Number of sheets = ____________________

Bill asked the vocational instructer if he had any material on carpentry that he might study to find applications of geometry on his own. On the next pages are some of the situations Bill came up with.

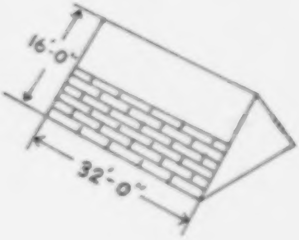
7. Figure the number of square feet of form work required for the end of the concrete retaining wall shown here (find the area of the trapezoid).
Sq. ft. ____________________



8. A carpenter is to construct a gable-roofed shed. The section has the shape of a triangle with a base 24 feet wide and base angles of 30° and 45° respectively. Let 1 inch equal 12 feet and construct this section.

How long are the rafters? ____________________ and ____________________

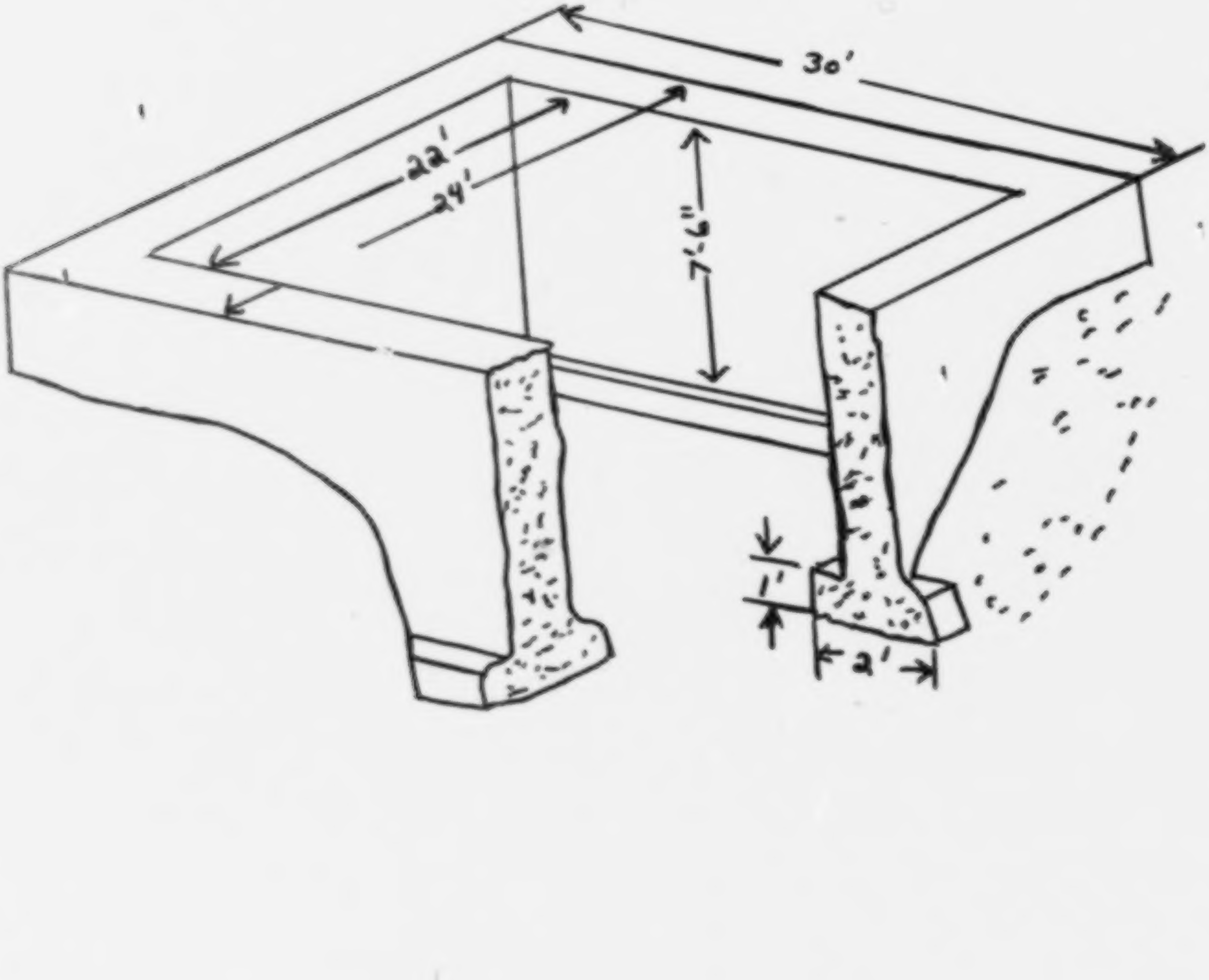
9. Figuring 100 square feet to a "square" of asphalt shingles, how many squares of asphalt shingles are needed to cover the roof shown here (make no allowance for waste)?



10. The basement of a building 30 feet long and 24 feet wide is to be excavated to a depth of 6 feet Find the number of cubic yards of material to be removed. ____________________

11. A basement foundation is shown below. One corner is removed showing a cross section through the wall. The broad base upon which the wall rests is the footing. The foundation is 30 feet long, 24 feet wide. The wall is 1 foot thick and 7'-6" high. The footing is 2 feet wide and 1 foot thick. Find the number of cubic feet in the walls and footings.

Volume = ______________________________



II-8 Architecture and Drafting

DRAFTING AND ARCHITECTURE

"Draftsmen translate the ideas, rough sketches, specifications, and calculations of engineers, architects and designers into working plans used in making a product." The Occupational Outlook Handbook tells of the 310,000 draftsmen working mainly for private industry. Most of these people have attended a vocational school or junior college to receive their training but a few have acquired their skills strictly through on-the-job training programs.

Most draftsmen start out as tracers earning approximately $6,000 a year. They look to a good future because design and the problems connected with it are becoming more important as the years go by. They will need to upgrade their skills however, because modern means for producing drawings and new electronic equipment will eliminate some of the lower level tasks now done by some draftsmen.

The architect differs from the draftsman in numerous ways. Although some architects find themselves often doing work similar to that of the draftsmen, most of their work usually deals with the idea behind and the design of an item. They most often have had five or six years of college ahd have passed the requirements for licensing that the individual state has established.

An architect needs an interest and ability in technical problems and their solutions but in addition he needs to have interest and ability to sense and see artistic concepts well. These two areas do not always fit together in one individual and should cause the would-be architect to spend some extra time looking at his abilities and interests.

Because more training is required to become an architect, architects usually command more pay than a draftsman does. Most begin at $8,510 or $10,528 if they go to work for the federal government. Their working conditions are good and chance for advancement is great if they possess the qualities desired by employers.

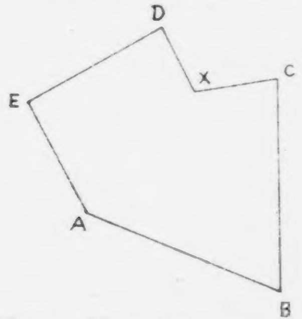
DRAFTING AND ARCHITECTURE

Draftsmen and architects are skilled technicians who work at a drawing board. Their jobs are to prepare working drawings of buildings, parts of buildings, or equipment for buildings. They take a designer's imagination and transform it into a builder's reality. They correlate the architectural, structural, mechanical electrical, and shop drawings with the building contractor's needs. To become a skilled draftsman or architect requires some background in mathematics. Let's take a look at some mathematics you will be able to apply in these two careers.

1. Construct a triangle having a perimeter of 8 inches and the ratio of sides 2:4:5. (Hint: See unit "Standard Constructions," problem 9.)

2. The hypotenuse of a right triangle is 4" and one angle is 30°. Construct the triangle.

3. To reduce a polygon so that the given sides become proportionately smaller is a common problem since most drawings are scale drawings. Reduce the polygon ABCXDE to a polygon so that the given side $\overline{AB}$ becomes $\frac{3}{4}$ inch.



Follow these directions:

a. Draw a given polygon ABCXDE.

b. Draw a segment A'B' parallel to $\overline{AB}$ so that the length of A'B' is $\frac{3}{4}$ inch. (A'B' can be drawn anyplace. It would be easiest to put it to the right of the picture. Just be sure it is parallel to AB.)

c. Draw AA' and BB' and extend these segments until they meet at a point we'll name O.

d. Join the points C, X, D, and E to O.

e. Through B' draw a line parallel to $\overline{BC}$ to meet $\overline{CO}$ at the point we'll call C'.

f. Through C' draw a line parallel to $\overline{CX}$ to meet $\overline{XO}$ at a point we'll call X'.

g. In the same way find points D' and E'.

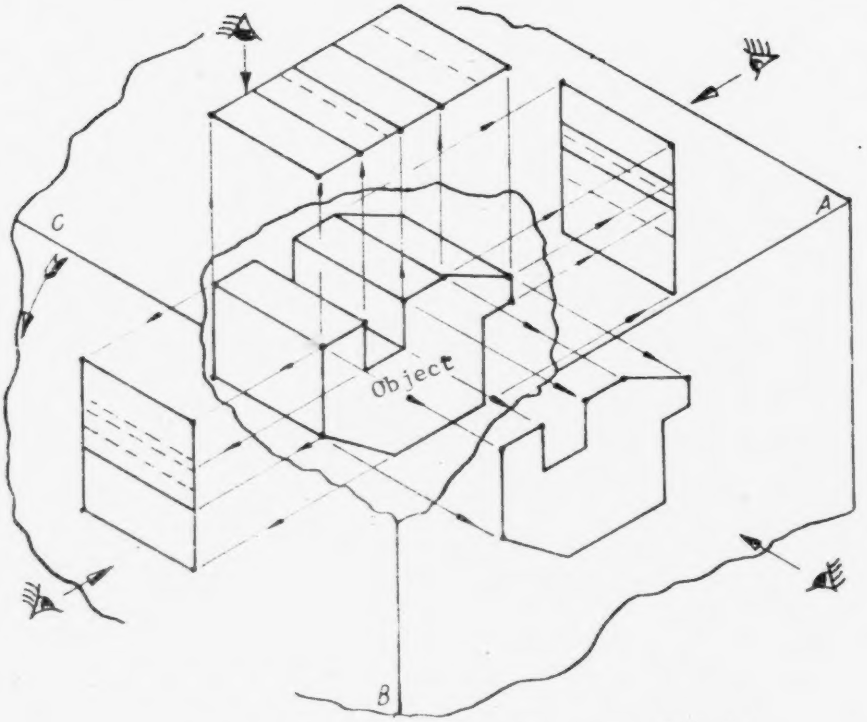
h. Now A'B'C'X'D'E' is the required polygon similar to ABCXDE.

In order to convey their ideas, architects and draftsmen must be able to fulfill two main requirements:

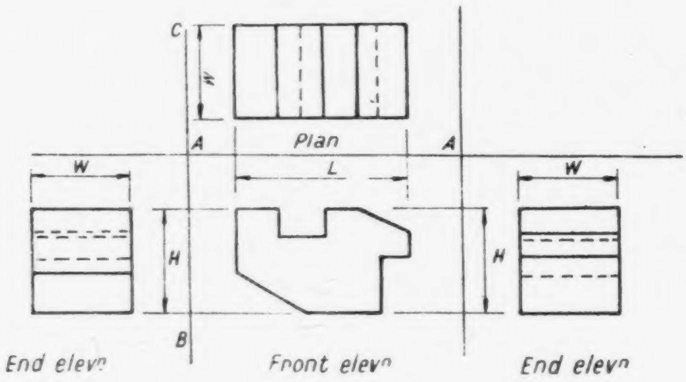
1. To give a clear and accurate picture of the shape of the object.
2. To show the size of the object.

DRAFTING AND ARCHITECTURE

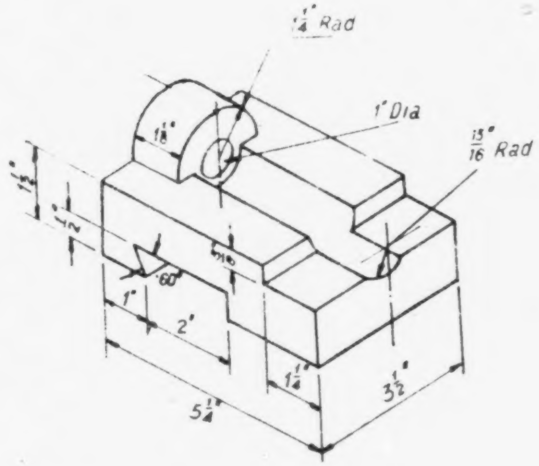
The drawings are a system of views showing separate drawings which give details in accurate proportion of the different sides and faces of the object. The object has three dimensions, but when drawn on a sheet of paper any one view shows two dimensions only. At least two projected views will normally be required to explain fully the shape and size of an object. The diagram below shows projected views from the front, back, and both sides as seen by the eye. Only the edges or outline of the object are shown as full dark lines; those edges that are not visible from any particular view are shown by dotted lines. The method you see illustrated is called "orthographic projection," and by using it many "projected views" of an object can be drawn to exact scale.



Order:- Eye - View - Object



4. Draw full size the front elevation (view), end elevation (view), and plan of the details shown in the object below. (Use the example just discussed as a model of what to do.) Write the dimensions in each drawing.



5. A draftsman is required to draw a circle for a pulley 2'-9" in circumference. What radius should he use? (Use $\pi = 3.14$).

Radius = ____________________

6. A special electronic device requires that 102 holes be equally spaced on a circle whose diameter is 2.75".

   a. What is the central angle between the holes? ____________________

   b. What is the distance on the circumference between holes to four decimal places? ____________________ (Hint: Make calculations from center to center of the holes.)

   (Hint: See unit "Circle Relationships" about length of an arc.)

7. A rectangular building 200' x 135' is drawn to a scale of $\frac{1}{8}$" = 1'-0". What is the size of the rectangle that will represent this building on a blueprint? Size = ______________________

8. The wing span of a jet plane is 56'. What will be the wing span of a model of this plane built to a scale of 1" = 8'-0" Span = ________________

9. Draw a plan to a scale of $\frac{1}{4}$" = 1'-0" to represent a room 20'-0" long by 16'-9" wide.

II-9 Optical Technician

OPTICAL TECHNICIAN

"Over half the population of the United States over the age of six wears eyeglasses." This may be surprising but our concern for good eyesight has made it true. It is becoming increasingly important for us to find quality eyewear.

After an examination has determined that glasses are necessary and a prescription has been written, two people have a great deal to do with the glasses. The first person is usually called the optical mechanic. He grinds the glass so that it will do what it must for the eyes. He also fits the lenses into the chosen frames. A dispensing optician then fits the eyeglasses to the face. Often an optical technician will perform both these duties.

Throughout the country, most people learn their trade on the job. Recently, however, many trade schools offer programs. Training is available at both Anoka and Eveleth. These programs last for a little less than a year and include such courses as grinding, theory, lens hardening, anatomy of the eye, and frame repair.

Many prospective technicians at the area trade schools never complete the course. There is such a demand for trained optical technicians that jobs are offered before the whole program is completed. When they do begin work, technicians earn a wide range of wages, depending upon skill, responsibilities, and location; usually, however, hourly wages run from $2.50 to $4.25 a hour. Purchasing one's own business opens up greater possibilities for increased income.

The mathematics required on the job is rather extensive. The use of tables makes the job much easier, but a thorough understanding of certain concepts of geometry and algebra are essential. It is difficult to extract the math used by the optical technicians and put it in meaningful career-related setting.

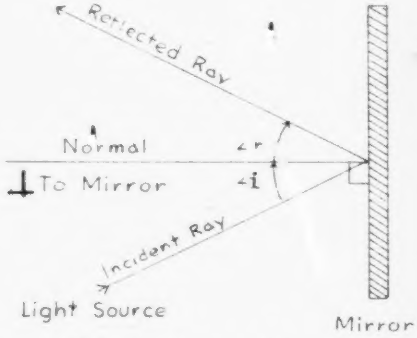
OPTICAL TECHNICIAN

There is much background information needed to lead up to the mathematics applications. The geometric concepts covered in the remainder of this unit are typical of those used in preparing for the job and on the job.

Light and its reaction to glass must be studied and understood. When a light hits a mirror, the angle of reflection is equal to the angle of incidence. The angle of incidence is formed by two rays: first, the ray of light hitting the mirror, and second, a ray perpendicular to the object the light is hitting drawn from the point where the light hits. See figure 1.

fig. 1

fig. 1



Reflection from a plane mirror
< i = angle of incidence, < r = angle of reflection

1. So if ∠i = 35°, the angle of reflection, ∠r, is ____________.

2. If we wished to bounce a light "around" a corner (see figure 2) to point A from any point B, what would the angle of incidence be? ______________

   Draw point B where this would occur.

fig. 2

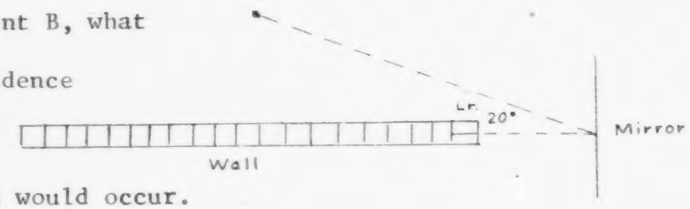
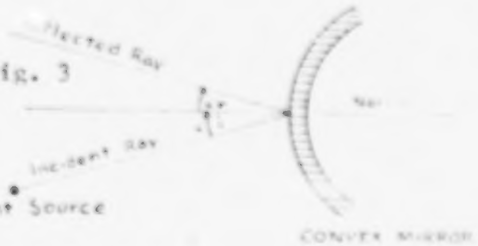
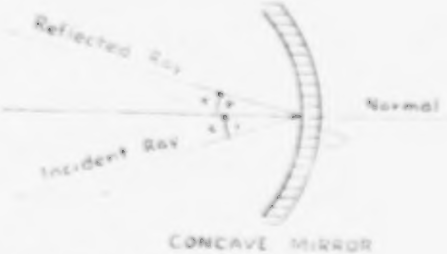


Fig. 3 shows that the reflected ray is the same as the incident ray on convex and concave mirrors. The light ray hits at the center, or apex of the curve in these examples. So, even though the mirror is curved, the light will be reflected at the same angle it comes in.

fig. 3



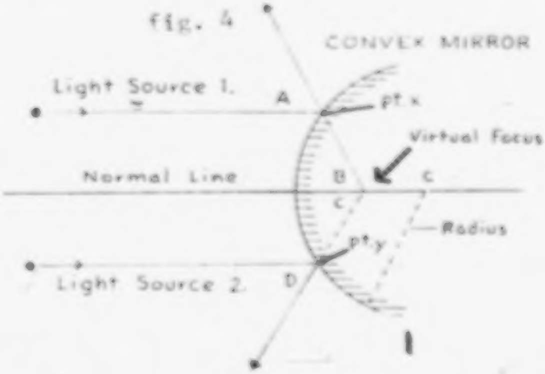
Reflection from a convex mirror.



Reflection from a concave mirror.

The question then arises, "But what if the light does not hit at the exact center? Surely it will then be affected by the curvature of the mirror." The answer is, yes it will. But the effect is a geometric arc; therefore we can predict what course the light will take after hitting the mirror. Figure 4 shows the geometric relationship between incoming light and reflected light on a convex mirror. Let's analyze how A is formed. Light source emits a ray that hits at point X. The light ray is parallel to the NORMAL LINE. At a point called the virtual focus (a point midway between the surface of the mirror and the center of its curvature), $\angle$ B is found by drawing a line from the virtual focus to point X.

fig. 4



3. Is $m\angle\bar{B} = m\angle A$? ____________________

If so why? ____________________

Notice that A makes the reflected ray diverge (go outward). Light source 2 hits at point Y. The ∠ D is formed by a ray from the virtual focus through D.

4. If m∠ C = 60°, at what angle does light source 2 reflect from the mirror?______________________________

Figure 5 shows the reaction of light as it hits a concave mirror at a point other than the center. Notice the difference in the paths of reflected light on the concave mirror as opposed to the convex mirror.

5. The convex had divergent rays that went outward, the concave mirror's reflected rays go ________________________ and are convergent at point A.

fig. 5

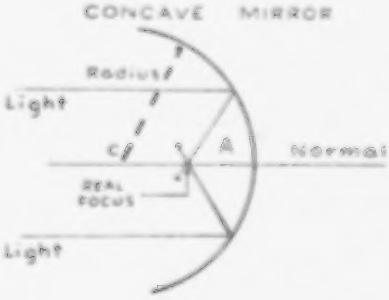
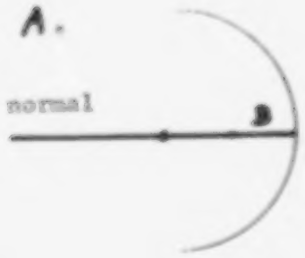


fig. 6



Point B on figure 6 is the point of convergence.

6. If a light source stems from a point A parallel to the normal line, what is the angle of reflection formed at the mirror's surface?_____________ (Draw the angle in figure 6; call the point where the light hits the mirror point D)

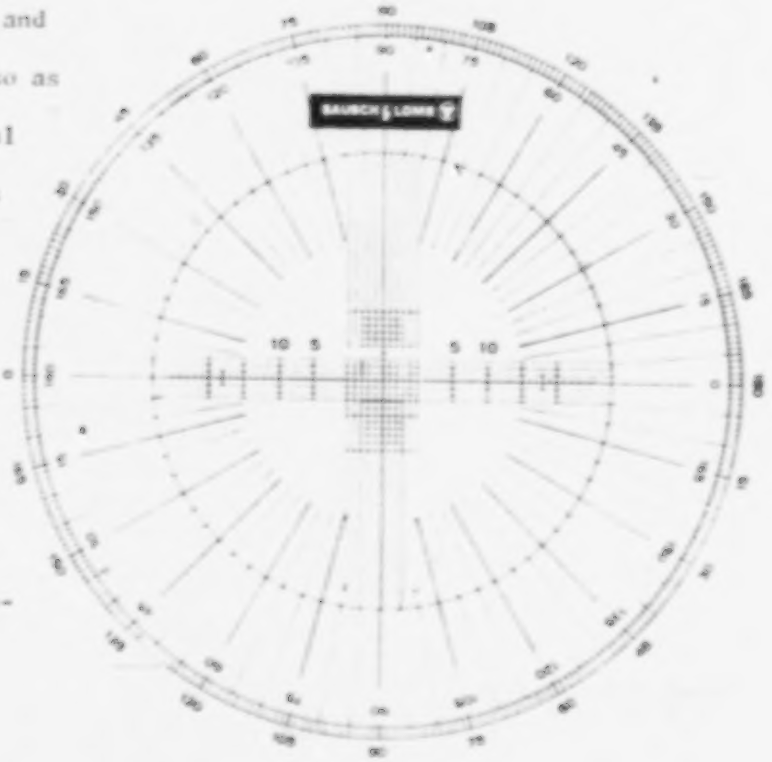
OPTICAL TECHNICIAN

7. Locate point C below the normal line whose reflection would form this same angle to point B.

An instrument used extensively in the layout and checking of lenses is the protractor shown below.

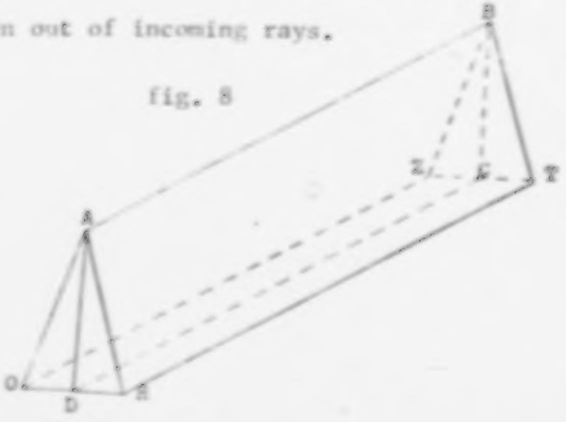
Reference points are the horizontal and vertical axes. A lens is laid out so as to obtain the vertical and horizontal placement of the optical center. As you can see, this protractor works on the same principle as the one used in construction of ordinary geometric figures; therefore, anyone interested in the optical technician field would have a good background if he knew how to use the protractor used in geometry class.

fig. 7



In optics, prisms are used to displace or change the direction of light coming into them. Certain types of light can be taken out of incoming rays.

fig. 8



8. With reference to fig. 8, list the faces of the prism. ____________

____

9. List the base of the prism (all light is refacted towards the base of a prism). ______________________ A base is defined to be any plane perpendicular to the base-apex plane (plane ABCD in figure 8).

10. Plot and label (using IJKL as points) another base (other than ORTZ) for the prism

As you can see, an optician must know the qualities of angles, a protractor, and prisms as well as right triangle relationship and many other geometric properties we have omitted.

II-10 Sheet Metal

SHEET METAL

Some geometry students were considering careers as sheet metal workers. They were concerned about the good they would get out of a course like geometry. Could they ever use it later? A sheet metal instructor was invited to the geometry class to discuss this problem.

He presented information of which most of the students were not aware:

> "Sheet metal workers usually need to complete a four-year apprenticeship program before they can become journeymen.
>
> "This apprenticeship training includes on-the-job and classroom training.
>
> "An apprentice receives between 40 and 80 percent of journeyman pay.
>
> "Average pay received by sheet metal worker in 1970 was $6.75 per hour.
>
> "As construction rates remain high, so will the need for sheet metal workers.
>
> "Sheet metal workers need to be able to picture, in their mind, what the finished article will look like (spacial aptitude).
>
> "Almost all sheet metal workers in the cities are union members.
>
> "The vocational-technical schools of Duluth, St. Cloud, Minneapolis, and St. Paul have programs in sheet metal that last from 18 to 21 months."

In addition to hearing the sheet metal instructor, a few students had their math teacher help them set up a tour of a local sheet metal shop. They went primarily to find out what kinds of geometry are needed on this job, but in addition they discovered that sheet metal workers need other abilities. For instance, much time was spent reading blueprints and preparing layouts so the flat sheets of cut material could be turned into three-dimensional objects. Here is where the ability to visualize a finished product became important.

It didn't take long for the geometry students to see that these workers need to work with some pretty interesting math concepts. Here are some typical problems a sheet metal worker might need to solve.

1. A cylindrical smoke stack made by bending rectangular sheets of tin is to have an inside diameter of 12 inches and a height of 10 feet. What is the surface area of the smokestack? (The formula for the lateral area of a circular cylinder is $A = 2\pi rh$ where h is the height.) How much material will be necessary if the sheets of tin are 14 inches wide by 10 feet in length and if two inches are allowed for overlap? a. ______________________________

   b. __________________ (Hint: Use $\pi = 3.14$) Round up any fractional part of a sheet.

2. You are required to make a tank within the following limits: it must be six feet high and contain 400 gallons of water (1 gallon = 231 cubic inches). What must the diameter of the tank be? Diameter = a. ______________________________

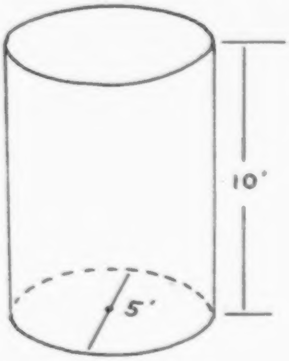
   How many square feet of sheet metal are required to make this tank, assuming no overlap? (Remember the formula for area of a circle is $A = \pi r^2$.)

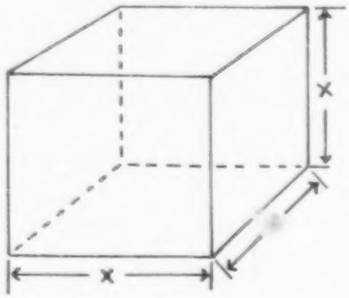
   b. ______________ square feet (Use $\pi = 3.14$) (It would be helpful to use a caculator.)

3. What is the capacity of a tank 10 feet long having an inside diameter of three feet? Capacity = a.__________________________

What are the dimensions of a cubical tank of the same capacity (1 = w =

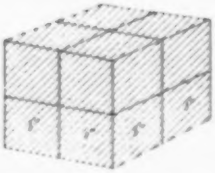
Use the cube root table attached. b.______________________________________





In the same way that a square root can be represented as the side of a square, a cube root is the side of a cube. The symbol for cube root is $\sqrt[3]{}$. To find the cube root of a number, we use the table shown. You will notice from the table that there are only five perfect cubes from 1 to 149. These are:

Perfect cube	Cube root ($\sqrt[3]{}$)
1	$1 = \sqrt[3]{1}$
8	$2 = \sqrt[3]{8}$
27	$3 = \sqrt[3]{27}$
64	$4 = \sqrt[3]{64}$
125	$5 = \sqrt[3]{125}$



THIS CUBE MEASURES 2 INCHES ON EACH SIDE. IT CONTAINS (2)(2)(2) = 8 CUBIC INCHES

$\sqrt[3]{8} = 2$

Table of Cube Roots (1 to 149)

	0	1	2	3	4	5	6	⑦	8	9
0		1	1.26	1.44	1.59	1.71	1.82	1.91	2	2.08
1	2.15	2.22	2.29	2.35	2.41	2.47	2.52	2.57	2.62	2.67
2	2.71	2.76	2.80	2.84	2.88	2.92	2.96	3	3.04	3.07
3	3.11	3.14	3.17	3.21	3.24	3.27	3.30	3.33	3.36	3.39
4	3.42	3.45	3.48	3.50	3.53	3.56	3.58	3.61	3.63	3.66
5	3.68	3.71	3.73	3.76	3.78	3.80	3.83	3.85	3.87	3.89
6	3.91	3.94	3.96	3.98	4	4.02	4.04	4.06	4.08	4.10
7	4.12	4.14	4.16	4.18	4.20	4.22	4.24	4.25	4.27	4.29
8	4.31	4.33	4.34	4.36	4.38	4.40	4.41	4.43	4.45	4.46
9	4.48	4.50	4.51	4.53	4.55	4.56	4.58	4.59	4.61	4.63
⑩	4.64	4.66	4.67	4.69	4.70	4.72	4.73	4.75	4.76	4.78
11	4.79	4.81	4.82	4.83	4.85	4.86	4.88	4.89	4.90	4.92
12	4.93	4.95	4.96	4.97	4.99	5	5.02	5.03	5.04	5.05
13	5.07	5.08	5.09	5.10	5.12	5.13	5.14	5.16	5.17	5.18
14	5.19	5.20	5.22	5.23	5.24	5.25	5.27	5.28	5.29	5.30

$$\sqrt[3]{\underbrace{10}_{\text{Left column}}\underbrace{7}_{\text{Top line}}} = \underbrace{4.75}_{\text{Value in table}}$$

If the number is between 1 and 149, but does not appear in the table, approximate the answer as shown in the following examples.

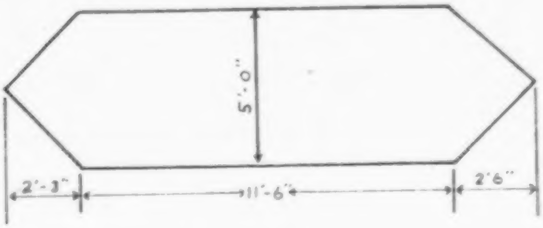
EXAMPLES:

1. Find $\sqrt[3]{121.7}$ to one decimal place.
   The table shows that $\sqrt[3]{121} = 4.95$ and $\sqrt[3]{122} = 4.96$. Thus $\sqrt[3]{121.7}$ is nearly 4.96, since 121.7 is closer to 122 than it is to 121. The answer is $\sqrt[3]{121.7} =$ 4.96 approximately. (Note: Approximations made by use of this table will be less accurate for cube roots of numbers from 1 to 20 than for numbers from 21 to 149.)

2. Find the side of a cube whose volume is equal to 142.35 cubic inches.
   side = $\sqrt[3]{142.35}$
   Using the table, we find that $\sqrt[3]{143} = 5.23$ and $\sqrt[3]{142} = 5.22$
   Therefore, S = $\sqrt[3]{142.35} = 5.22$", approximately.

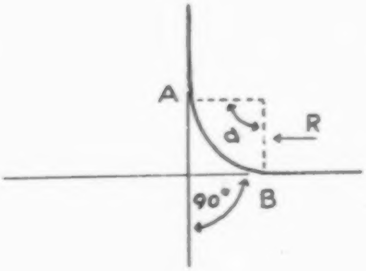
SHEET METAL

4. How many square feet of sheet tin are required to cover the following shape? ____________________



5. A sheet metal worker makes a form in the shape of a right triangle. The larger acute angle is 40° more than the smaller angle. Find each of the acute angles. (Either use the fact that "the sum of the angles equals 180°", or that "the two acute angles of a right triangle are complementary.") The measures of the angles are ________________ and ____________________.

6. Find the length of an arc in a piece of sheet metal bent to a radius of 3" at an angle of 90°. See figure below.



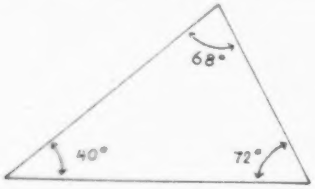
angle of a°

radius of R

The angle of 90° is the central angle.

length $\overset{\frown}{AB} = \frac{a}{360} \times 2\pi R$ where the circumference of the cirle with radius R is $C = 2\pi R$. Length of arc = ________________________

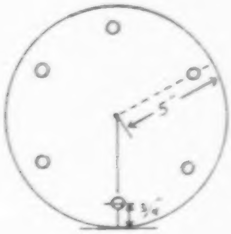
7. A sheet metal worker is asked to cut a piece of copper in the shape shown.



When he starts to make his layout, he finds that he needs more information. What is the least amount of additional information he needs to make the layout?____

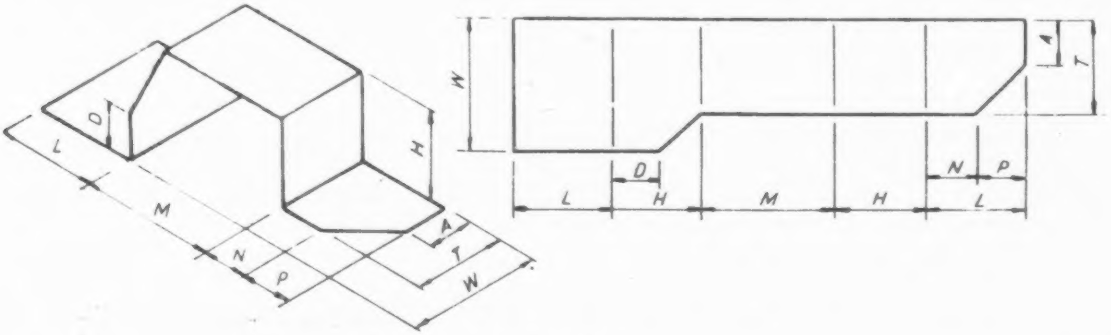
____

8. On a circular piece of sheet metal lay off six equally spaced ¼" screw holes with centers $\frac{3}{4}$ " from the circumference of a 5" diameter circle. See sketch below. (Draw a circle on paper and do this)

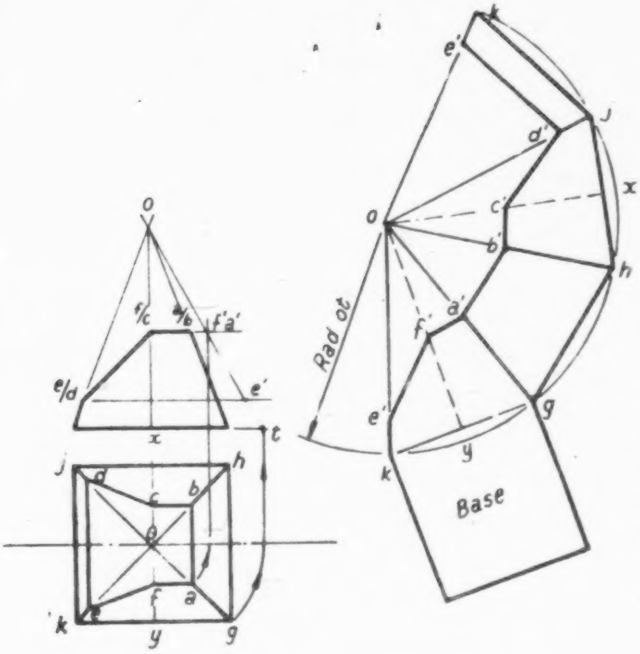


SHEET METAL

Many manufactured articles are produced in sheet metal from pressings and blankings, which are then joined at the seams or edges to form the completed component. To estimate the cost, it is important to be able to find the exact shape of the surface area of the complete part. To do this the object is "developed", i.e. it is drawn unfolded along the joints and "fold" lines, giving the exact shape of the area of metal sheet required to make it. To indicate the drawing construction it is usual to mark the given views (called orthographic views) at various points (using letters or numerals), so that they may be properly related to the"development drawing." The following shows the development of a sheet metal bracket and the frustrum of a square based pyramid. Remember that only true lengths can be used for the development, and be sure that these have been derived correctly from the given views.



Sheet Metal Bracket



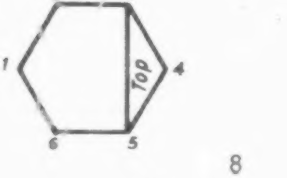
SIMPLE DEVELOPMENT

Allowance for folding, overlap for seams and joints etc not shown.

Joint line should always be taken on the shortest length. Developments must use true length lines not "projected" lengths.

Frustrom of a square based pyramid.

9. Draw a development of the frustrum of a hexagonal prism shown below. To do this the true length of each face of the hexagon is shown in the lower plan (distance 1 to 2 for example). The hexagon is made up of six panels which may be developed alongside the elevation (top plan), the height of the prism should be transferred by direct projection of lines. The base and the uncut portion of the top are then added to complete the development layout.



II-11 Engineering

ENGINEERING

Engineering, in 1970, had nearly 1,100,000 people employed, making it the second largest professional occupation. Teaching is the only profession with more employed than engineering. Most of these engineers work in some area of the manufacturing industry; the others work in governmental agencies and schools.

Usually, an engineer is required to attend school for four to five years. This time is spent studying both the basic sciences and the social sciences with the last half of the program emphasizing the student's engineering specialty. He becomes licensed by taking and passing a rigorous examination.

Some engineers have recently found themselves out of work. This has been due, in part, to the fact that defense spending dictates some of the needs for engineers in manufacturing. If government spending changes or drops off, the employment of many might be affected. In addition, our economic conditions will affect the need for engineers.

In 1969-1970 a first-year engineering graduate with no experience earned $10,400. If well-prepared and doing a good job, engineers' earnings can increase rapidly. Present engineers with 21 to 23 years of experience earn an average salary of $18,350.

A few of the larger branches of engineering are:

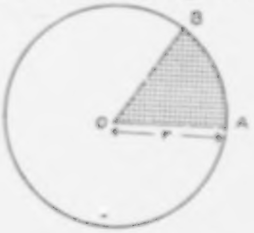
1. Electrical engineer
2. Civil engineer
3. Mechanical engineer
4. Aerospace engineer

ENGINEERING

5. Chemical engineer
6. Metallurgical engineer
7. Ceramic engineer
8. Agricultural engineer

Most engineers specialize in one of the many branches of the profession. Nevertheless, the basic knowledge required for all areas of engineering often makes it possible for an engineer to shift from one field to another. An engineer might encounter the following problems:

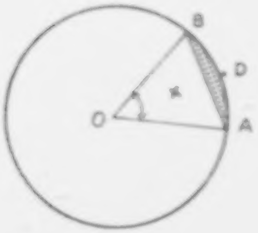
The area of a sector of a circle is equal to a fraction of the area of the circle determined by dividing the size of the angle by 360.



Sector AOB of X° in a circle of radius r.

Area of sector AOB = $\left(\frac{x}{360}\right) \cdot \pi \cdot r^2$

The area of a segment ABD (the shaded portion of the figure below) equals the area of sector AOB minus the area of triangle AOB.



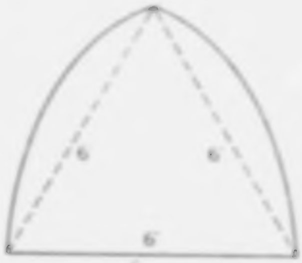
Find the area of Δ AOB by the formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ where a, b, and c are lengths of the sides and $s = \frac{1}{2} \cdot (a+b+c)$. If you know the altitude of Δ AOB, then use the formula $A = \frac{1}{2} \cdot b \cdot h$. In some cases, you might have to find the area of Δ AOB using trigonometry.

ENGINEERING

Consider the following situations:

1. A gothic arch is formed by two arcs, each one sixth of a circle. The center of each circle is at the ends of the width of the arch. The radii of the two arc sides equals each other and the width of the arch. (See the diagram.) Find the area of the gothic arch of radius 6 ft.

   Area = ____________________



2. Running down the center of a four-lane highway is a safety guard made of concrete. The base of the cross-section of the guard is a trapezoid and the top is a semicircle. The lower base of the trapezoid is 18 inches; the upper base (which is also the diameter of the semicircle) is 6 inches; and the distance between the bases is 12 inches.

   a. Draw a sketch of the cross-section showing the dimensions.

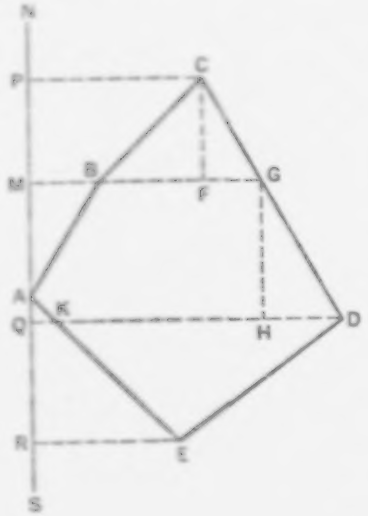
   b. How many cubic yards are there in one mile of the safety guard?

      Cubic yards = ________________________

ENGINEERING

3. A solid cylindrical pillar of iron supporting a building is 14 feet high and has a diameter of 9 inches. If it weighs 450 pounds per cubic foot, its weight is about __________________pounds. (Use 3.14 = π.)

4. A field has the shape of the irregular polygon shown in the figure. AM = 45 rods; PC = 35.5 rods; QD = 64 rods; RE = 30 rods; AQ = 6 rods; AR = 30 rods; angle FCG = 30°. Find the area of this field in square yards. (1 rod = 16½ feet).



Area = ________________________square yards.

5. An officer of engineers, mapping a piece of land lying between a straight road and a curved stream, obtains the data given in the table on the following page. This table shows that at various points from one end of the road the distance from the road to the stream is s yards and the distance from the point on the road to the end of the road is d yards.

   a. Using as a scale 1/8" = 1 yard, draw a map of the land. (Be sure to read the following hint before attempting.)

   b. Find its approximate area.

s	0	17	56	109	174	200	184	132	96	60	0
d	0	10	20	30	40	50	60	70	80	90	100

Hint: In actual practice, a large number of surfaces are enclosed by curves, for, example the field AFKD, bounded on one side by a river:

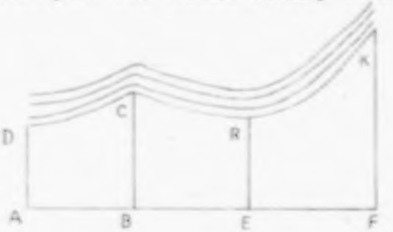


Fig. A

To find its area, first subdivide the field into convenient sections as shown by the lines CB and RE. The area of the plane figure bounded by a curve such as ABCD can be approximated from an accurate drawing of the figure by finding the average height of the curve and multiplying this height by the base. By the "average height of the curve" we mean the height of a rectangle equal in area to the area of the plane figure with the same base.

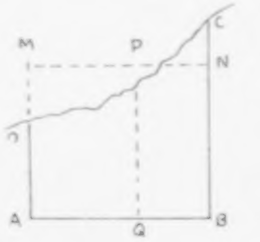


Fig. B

If you consider the section ABCD of figure A above, PQ is the average height of the curve from base AB.

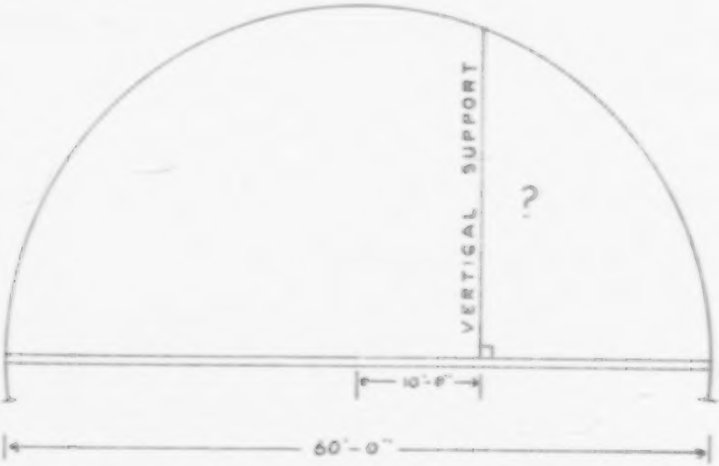
The average height may be approximated by drawing the horizontal line MPN so that the area DMP, which is added to form the rectangle ABNM, is as closely as possible equal to the area CNP which is cut off to form the rectangle. Since the area added is approximately equal to the area cut off, the areas of the rectangle ABNM and of the plane figure ABCPD can be considered equal (for all practical purposes)

a. $\frac{1}{8}$" = 1 yard

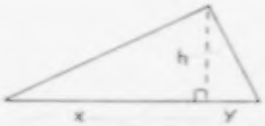
b. Approximate area = ________________

6. The dome of a building is in the shape of a semicircle that spans 60'. What is the length of the vertical support 10; to the right of center.

Vertical support = ____________________feet.



Remember that the length of the altitude of a right triangle, h, is the mean proportional between the two segments, x and y, that it divides the hypotenuse into.

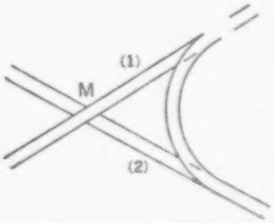


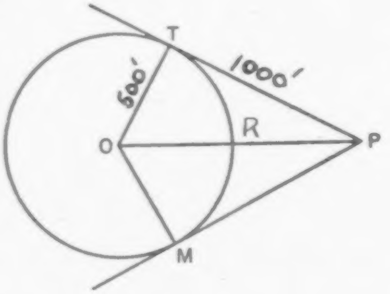
$$\frac{x}{h} = \frac{h}{y}$$

ENGINEERING

7. Suppose that two straight railroad tracks (1) and (2) intersecting at a point M, are to be connected by a circular track. The circular track is to be placed tangent to tracks (1) and (2). Assume that it is a distance of 1000 feet from the point of tangency to the point of intersection of the two straight tracks. Also, suppose the radius of the circular track must be 500 feet. How far will the intersection of the two straight tracks be from the circular track? ____________________

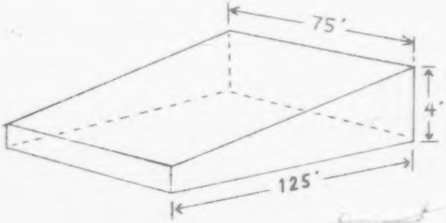
(In other words, find the length of PR.)





b. What is the relation between $\overline{OT}$ and $\overline{TP}$?

8. How many cubic yards of soil will be required to fill a section of land 75 feet wide by 125 feet deep if the surface is to be raised four feet in the rear and gradually sloped to the front, where it is to be one foot deep?



9. In building a bridge, panels as pictured below are needed. In the figure, the timbers CB, DF, and EG are perpendicular to CA. From the given dimensions find the lengths of the timbers.

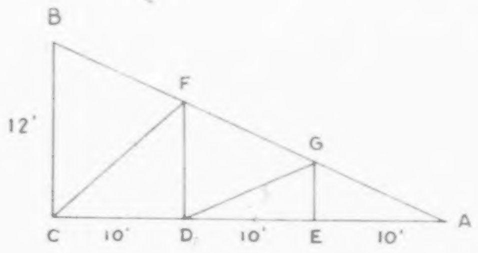
a. DF = ____________________

b. EG = ____________________

c. CF = ____________________

d. DG = ____________________

e. AB = ____________________



Hint: Use proportions from corresponding sides of similar triangles.

II-12 Machinist

MACHINIST

Alex heard that the counselor at his school would go over his school record with him so he made an appointment. While talking with him, Alex thought about how he could use all of this information about himself. The counselor thought this information should be used as Alex planned his future.

After the interview, Alex decided to put down on paper some of the things that he had learned about himself. He listed these items.

My grades are best in shop courses.
My interest lies in the mechanical area.
An aptitude test shows I have highest ability in spatial and mechanical areas.
My favorite hobby has always involved the building of things.
I once won an award for a project I made in machine shop ...

"Machine shop! That was my favorite class. Maybe a career in the machining trades would be fun." Alex went back to the counseling office the next day to look up some information about this area.

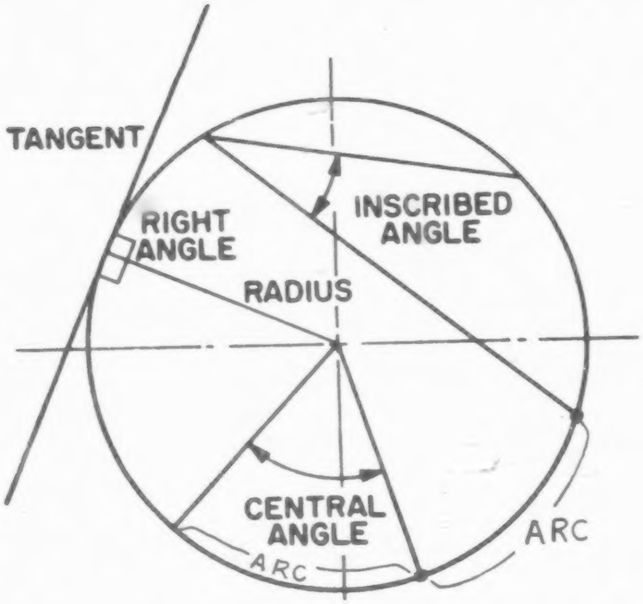
In the Occupational Outlook Handbook, Alex found there are at least five specific jobs in the larger category of machining occupations. They are: machinists, tool and die makers, instrument makers, machine tool operators, and setup men. Each of these jobs requires certain specific skills that make them different from one another, but there are many similarities, too. All of these jobs usually involve operating power-driven machinery; they require accuracy and that accuracy involves precision handwork.

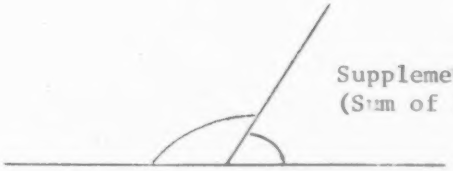
The handbook continued, "All-round machinists are skilled workers who can operate most types of machine tools. Machine tool operators commonly operate only one kind of machine tool. Tool and die makers specialize in making dies for use with presses and diecasting machines, devices to guide drills into metal, and special gauges to determine whether the work meets specified tolerances. Instrument makers use machine tools to produce highly accurate instrument parts made of metal or other materials. Setup men adjust machine tools so that semiskilled machine tool operators can run the machines."

Alex found that the most common way to enter these trades is through an apprenticiship program of on-the-job training. For the machinist and tool and die maker, this training can take four or five years.

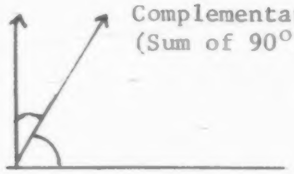
There was much information in the guidance office about these trades but one thing especially stuck in Alex's mind. It seemed that the future of these trades could depend much on how computerized or numerically controlled machines would be used. At least the machinist will be doing different things and will have to continue to improve his skills as the years go by. Machines that are computerized can do the entire job of turning out a product -- if instructed properly and maintained adequately. The people working with these highly complicated machines will indeed need special skills.

Alex found that uses of geometry in the machinist trade are an everyday occurance. Angle relationships, circle relationships, and polygons are the most frequently used topics. Some of these relationships are shown below. For more information on these relationships, refer to the "Polygon" and "Circle Relationships" units.

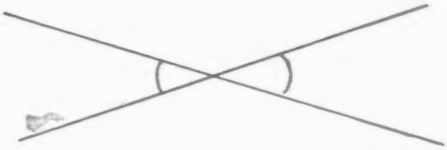




Supplementary angles
(Sum of 180°)



Complementary angles
(Sum of 90°)



Vertical angles
are congruent

Polygons

3 sides --- triangle

4 sides --- quadrilateral

5 sides --- pentagon

6 sides --- hexagon

7 sides --- heptagon

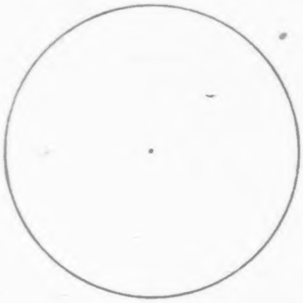
8 sides --- octagon

9 sides --- nonagon

10 sides --- decagon

MACHINIST

The next semester Alex decided to take an advanced machine shop class. One of the first projects was to machine, from a piece of round stock, a "hex" headed bolt. He knew that "hex" was short for hexagonal, meaning six-sided. He decided to take the 2" (2" diameter) and lay out the six sides on one end. After he cut the piece to the length he desired, Alex put it in a vise, end up. Before going any further he made a drawing of what he wanted to do.



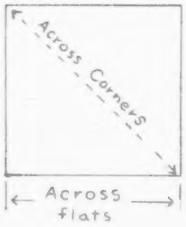
In order to get a six-sided figure in the circle, he had to divide it into equal parts. How may degrees would each part contain? 1. ____________________

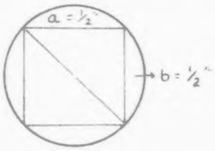
[Note: There are 360° in the circle]

Alex knew that the central angle (an angle whose vertex is at the center of the circle) is equal in measure to its intercepted arc. Show how he inscribed a hexagon in the circle provided above. After that, he ground the edges and cut the threads.

His next problem was to machine a $\frac{1}{2}$" (across the flats) square nut. In order to choose the correct round stock, he must find the length across the corners (diameter). Use the diagram of the square nut to find the "across corners" dimension. (Hint: $c^2 = a^2 + b^2$)

2. Across corners dimension: ______________________________

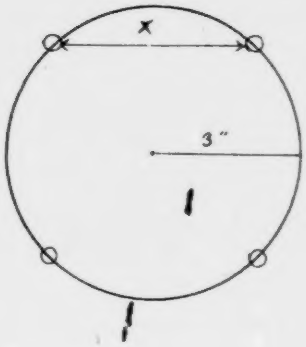




1" = $\frac{1}{2}$" scale

3. What is the distance between centers of the four holes equally spaced on a 6" circle?

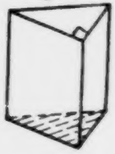
X = ____________________



For the next problems, you will need the given sine table at the end of the unit.

4. A right triangular prism (fig.1) must be machined out of round stock. Find the diameter of the stock needed; use fig. 2 (drawn actual size) to do your work.

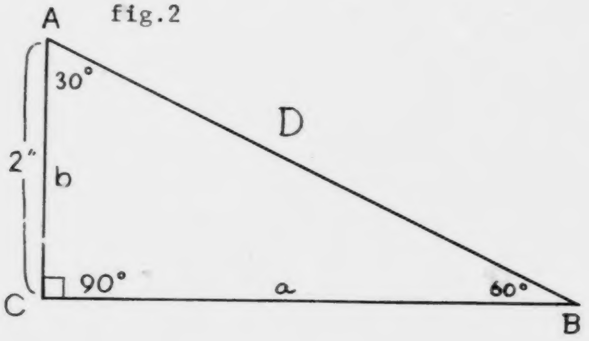
fig. 1



TRIANGULAR PRISM

a. Circumscribe a circle around △ ABC (Construct perpendicular bisectors of any two sides. Where they meet is the center of the circumscribed circle.)

fig.2



b. The diameter of this circle can be found by using the formula:

$$D = \frac{b}{\sin \angle B}$$

that is, diameter is equal to the length of segment b divided by the SINE of angle B. (Sine of an angle is the ration of the $\frac{\text{length of opposite side}}{\text{length of hypotenuse}}$)

Use the sine tables at the right to find the sine of B.

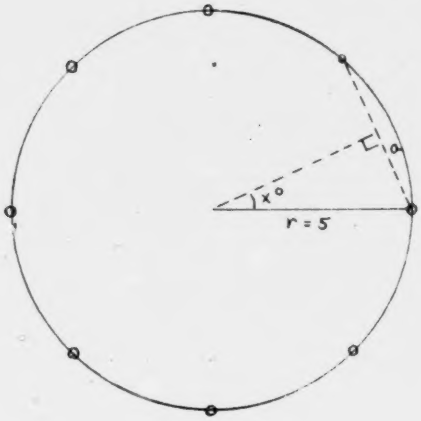
a. sin B = ______________

b. Find the diameter of the stock needed. (Find D)

Diameter = ______________________

Angle	Sine	Angle	Sine
1°	.0175	46°	.7193
2°	.0349	47°	.7314
3°	.0523	48°	.7431
4°	.0698	49°	.7547
5°	.0872	50°	.7660
6°	.1045	51°	.7771
7°	.1219	52°	.7880
8°	.1392	53°	.7986
9°	.1564	54°	.8090
10°	.1736	55°	.8192
11°	.1908	56°	.8290
12°	.2079	57°	[illegible]
13°	.2250	58°	[illegible]
14°	.2419	59°	.8572
15°	.2588	60°	.8660
16°	.2756	61°	.8746
17°	.2924	62°	.8829
18°	.3090	63°	.8910
19°	.3256	64°	.8988
20°	.3420	65°	.9063
21°	.3584	66°	.9135
22°	.3746	67°	.9205
23°	.3907	68°	.9272
24°	.4067	69°	.9336
25°	.4226	70°	.9397
26°	.4384	71°	.9455
27°	.4540	72°	.9511
28°	.4695	73°	.9563
29°	.4848	74°	.9613
30°	.5000	75°	.9659
31°	.5150	76°	.9703
32°	.5299	77°	.9744
33°	.5446	78°	.9781
34°	.5592	79°	.9816
35°	.5736	80°	.9848
36°	.5878	81°	.9877
37°	.6018	82°	.9903
38°	.6157	83°	.9925
39°	.6293	84°	.9945
40°	.6428	85°	[illegible]
41°	.6561	86°	.9976
42°	.6691	87°	.9986
43°	.6820	88°	.9994
44°	.6947	89°	.9998
45°	.7071		

5. Eight bolt holes are equally spaced on the 10" circle. What is the center-to-center distance of any two adjacent holes? (Hint: How large is a central angle? Draw appropriate perpendiculars.)



Step 1. Find the measure of a central angle.

Step 2. $X^{o} = \frac{1}{2}$ measure of a central angle.

Step 3. Look up the sine of X^{o}=(approximate $\frac{1o}{2}$), substitute into the formula. Sin $X^{o} = \frac{a}{5}$

Step 4. Solve for a

a = ______________

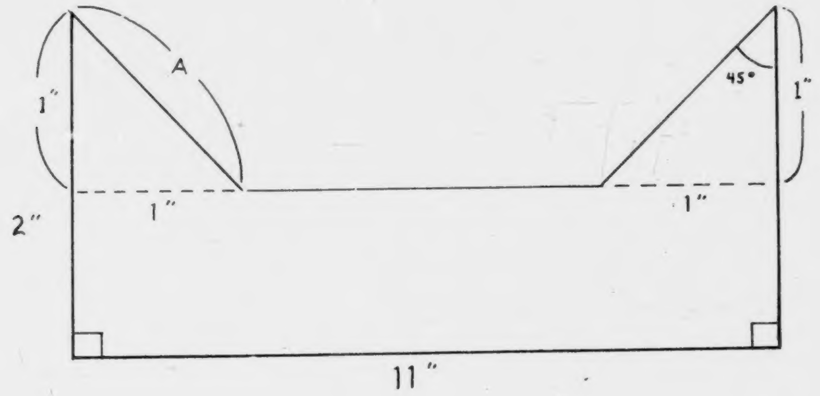
Step 5. Double a to get distance between centers of two adjacent holes.

2a = _________________

6. Another application of the sine function is to find the missing length of a side of a triangle given an angle and the length of the opposite side.

Find length A.

Hint: Sine $45^{\circ} = \frac{1}{A}$



II-13 Cement Worker

CEMENT WORKER

A cement worker or cement mason in often also called a concrete finisher. The name concrete finisher is probably the best one to use because it most clearly tells what work is done. In this area of work it is easy to see how specialization has set in. On large construction projects, the mason that does the whole job of putting in the concrete is gone. No longer does one man build form, mix and pour the concrete, and then finish it. On most projects, the concrete finisher may supervise some of the initial phases of the laying of concrete, but his main concern is the last steps which make the concrete look like it does when it hardens. Even this phase has various specialists, however. There are finishers that work primarily with large power equipment on big projects (highways, etc.) and there are others that work with mastic (a fine asphalt mixture that is applied over the concrete).

A large number of concrete workers have obtained their training on the job. These are people who work their way up as they gain experience and are able to do the job. This trade does, however, have a three-year apprenticeship program requiring the related classroom instruction.

Many workers will be going through the apprenticeship and the on-the-job programs as the next few years go by. This is because the demand for concrete workers is great because of the fast growth of the construction industry. If construction activity continues at its rapid pace, all types of trained building tradesman will be in demand.

As a cement mason, one could expect to earn $6.02 in 1970. The range of hourly wages ran from $7.80 down to $4.35 depending upon the city. In the rural areas a cement worker's wage also varies much from one locality to another.

CEMENT WORKER

The mathematics involved with cement workers varies with the particular phase of the job. The workers that go out to the job first and construct the forms for the job must have a good knowledge of measurements and geometry. First of all, these people must be sure that all work to be done is level or has the correct slope for proper drainage. If a design of any type is desired, they must be able to construct the forms for the desired effect.

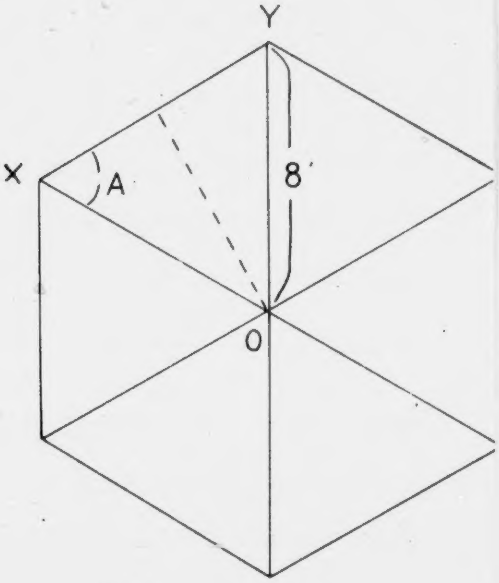
For example, if a patio with redwood spokes in the form of a hexagon is desired, the men must be able to determine the lengths of each side as well as the angle formed.

1. Find m∠A__________________________________

2. Find length of $\overline{XY}$__________________________________

3. Find the area of △ OXY - ______________________________

Hint: See the unit on "Polygons and Their Areas" or your teacher to get help in finding the area of △ OXY.



In this case, the men working on the forms also put in the order for the amount of concrete needed. If the patio is to be 4" deep, find the number of cubic feet (to the nearest $\frac{1}{2}'$) needed for this job.

4. Total area of patio (total area = area of one △ x number of △ 's)________________

5. Total volume in cubic feet (V = total area x depth -- change units to feet)

____

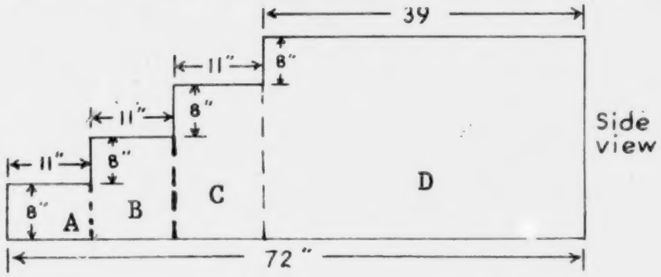
(Ask your teachers or read the unit "Volume" for help if you do not understand "cubic feet.")

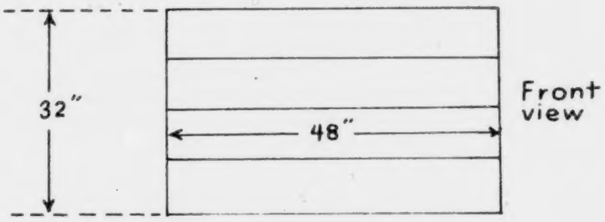
CEMENT WORKER

6. Total volume in cubic yards________________________

When the forms are completed, the concrete is ordered and poured. Finishers smooth the surface for a mirror-like finish.

Another common job for a cement contractor, especially the smaller ones, is to install front or rear steps. The small contractor will cut boards for forms to the specifications in the drawing, and then figure the amount of concrete necessary. Usually it is not a completely solid block of concrete; some fill is left on the inside. If the contractor figures that the steps shown in figure 2 will be one-third sand fill, how much concrete must he order for the following situation?





Hint: Find the total volume of the steps and subtract the fill.

a. Find total area of the side view (total of areas A, B, C, and D).

Total area = ________________________

b. Change area from square inches to square feet. A = ____________ sq. ft.

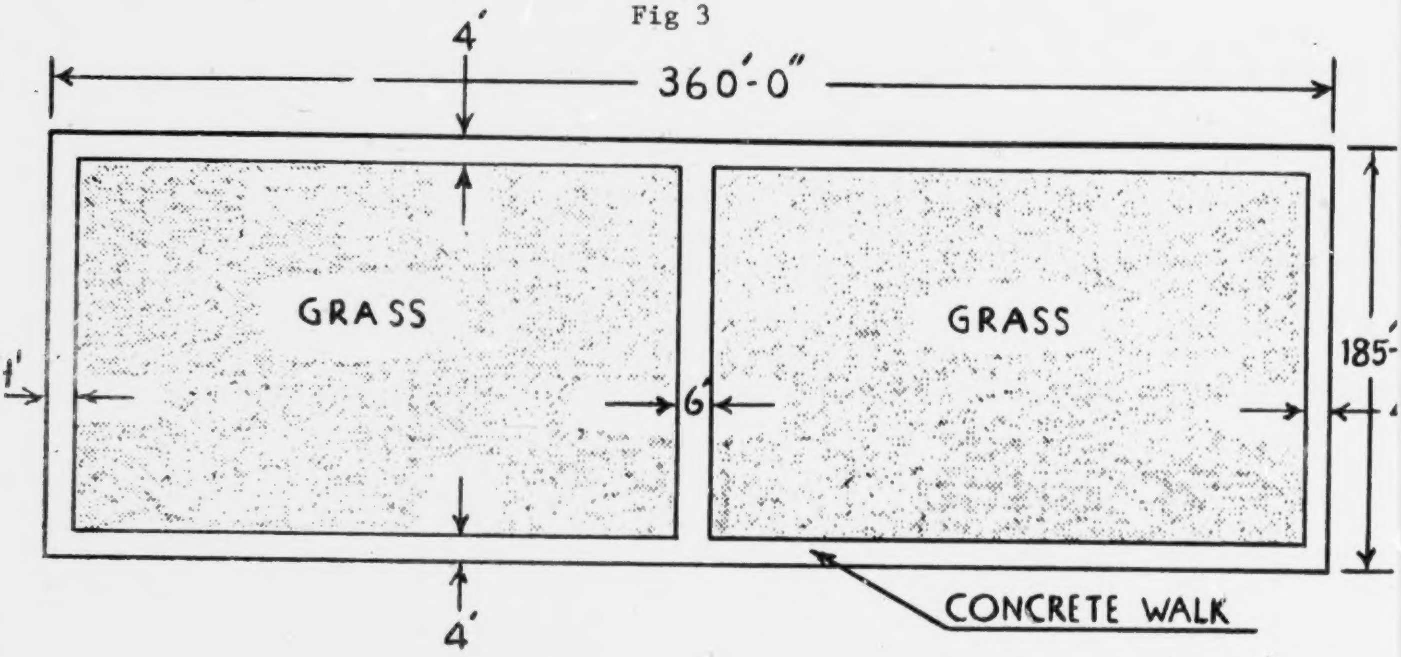
c. Find volume by: V = total area x depth (48"). V = __________ cubic feet.

d. Change inches to feet and multiply. Now subtract one third of the total from your answer. This will give you the amount of concrete that must be ordered in cubic feet. This must be changed to cubic yards. If 27 cubic feet equals one cubic yard, how many cubic yards must be ordered? __________

CEMENT WORKER

The G & P cement contracting company was fortunate enough to submit the low bid for the sidewalks around a proposed new park. The diagram of the project is figure 3.

Fig 3



8. If the sidewalks are to be 4" thick, how many cubic yards of concrete need to be ordered?______________________________

CEMENT WORKER

G & P also was contracted for a triangular area in front of a library. Two different ideas were discussed. One allowed for a 6' x 8' x 10' triangle with a large grass border. The other was for a similar larger triangle whose sides are three times the length of the first triangle. As your next project, make a scale drawing of each triangle. (Use $\frac{1}{4}$ " = 1' scale on the graph paper provided.) Find the cost differential of the concrete work between the two designs (assume G & P has figured a total of $35 per cubic yard for material and labor).

Refer to these two triangles as you work problems 9 through 13.

9. What is the minimum relationship for two triangles to be similar? ________________

10. What is the ratio between the sides of the two triangles? ________________

11. What is the ratio of the areas of the two triangles? ________________

    Hint: Use Hero's formula for area

    $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$ and a, b, c are the lengths of the sides of the triangle

12. From 11, how many times greater is the cost of the larger triangle than the smaller triangle? ________________

13. What rule can be deduced about areas of similar triangles? ________________

    ____

Those that work with cement can be put into two general classifications: the specialist -- one who works with a single phase of the operation, and the generalist -- one who works with the project from start to finish. Geometry is an integral part of both types, because each works with figures. The homeowner who wishes to save money on a home or patio or steps would hire a generalist who would need to use practical geometry.

II-14 Forestry

FORESTRY

Foresters are concerned with managing, protecting, and developing the nation's forests and wildlife areas. Their work covers a wide variety of activities, such as mapping the location of timber, estimating the amount of timber, supervising the harvesting and cutting of trees, and purchasing and selling timber. They may also be involved in safeguarding forests from fires and disease, wildlife protection, or managing camps, parks, or land.

You can see that much of a forester's work is done outdoors. The work is not always easy and requires that a person be in good physical condition. Likewise this work is often performed in remote areas -- a forester must not object to that. In addition, a "would-be" forester should have a strong interest in science, for many of the courses that he will be required to take involve scientific concepts.

These courses, as well as others, will be taken during college preparation which is required of all foresters. A major in forestry is required and a list of colleges offering this kind of preparation is available in the guidance office of your school. Generally, courses in forest resources are also required for the forestry major.

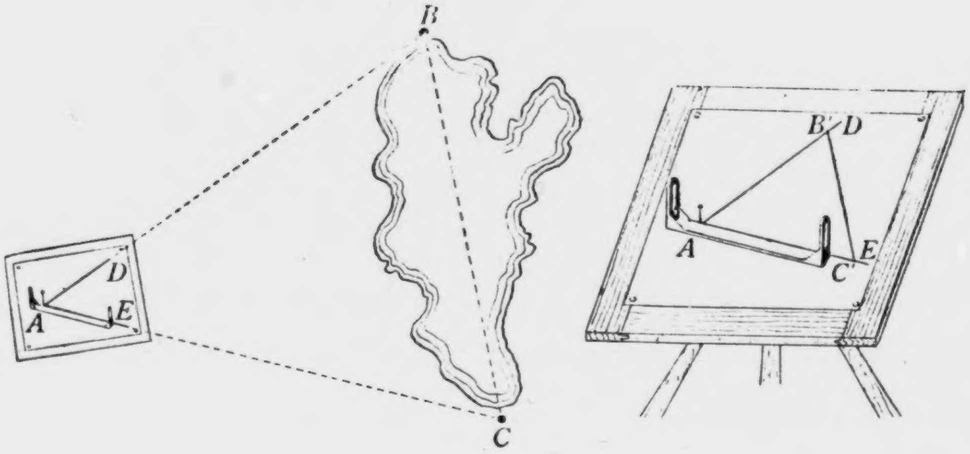
As is the case in most careers today, specialization is affecting the forester, too. More and more of the work now done by the forester will be done by a specialist. The Occupational Outlook Handbook says, "Specialized scientists: biologists, horticulturists, agronomists, chemists, etc., increasingly will be hired for the more scientific work previously performed by foresters." This means, of course, that the need for foresters may not increase as much as it would without these specialists. Government agencies (by far the largest employer of foresters) will continue to need them as they expand their services and as more land is put into recreational use.

Do you have an interest in nature and a desire to preserve it? Do you enjoy working outdoors in a job requiring physical exertion? Do you have an interest in science and in school so that you see yourself going to four years of college? If you answer "yes" to these questions, it may be that forestry is a possible career for you. Don't rule out forestry if you don't see yourself in college either -- there are forestry aides and technicians that don't have to have college training.

A forester's work involves many ideas and methods of indirect measurement as will be seen in some of the problems that follow. An instrument commonly used for field work in indirect measurement is the plane table, which consists of a table mounted on a tripod. By means of a small "level," the table can be set in a horizontal position. A small ruler with "sights" is arranged so that the "line of sight" is parallel to the edge of the ruler. An object is in the line of sight when it can be seen through the narrow vertical openings in the sights. The diagrams on page three show how a plane table is used to indirectly measure a distance, BC, which could not be measured directly because a pond was between the points B and C. (See figure 1.)

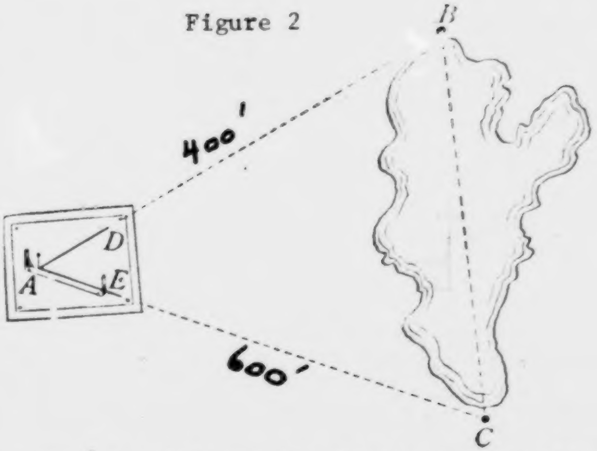
FORESTRY

Figure 1



1. Suppose we set up the table at a point from which B and C can be seen easily. After fastening a sheet of paper to the table top, put a pin in the board at A. A ruler is placed with one edge against the pin and revolved to point in the direction of B. A line AD is drawn along the ruler giving the direction of point B from A. The ruler is then revolved about point A again, to sight in the direction of C. Another line, AE, is drawn along the ruler, giving the direction of point C from A. Now the distances from A to B and from A to C are measured directly with a tape rule. For example, let the distances AB and AC be 400 feet and 600 feet respectively. (See figure 2.)

Figure 2



Choose a suitable scale, say $\frac{1}{4}$" = 25' and locate the points B' and C' on the plane-table drawing by marking off AB' and AC' to the scale on lines AD and AE respectively. (Mark AB' and AC' off on figure 3.)

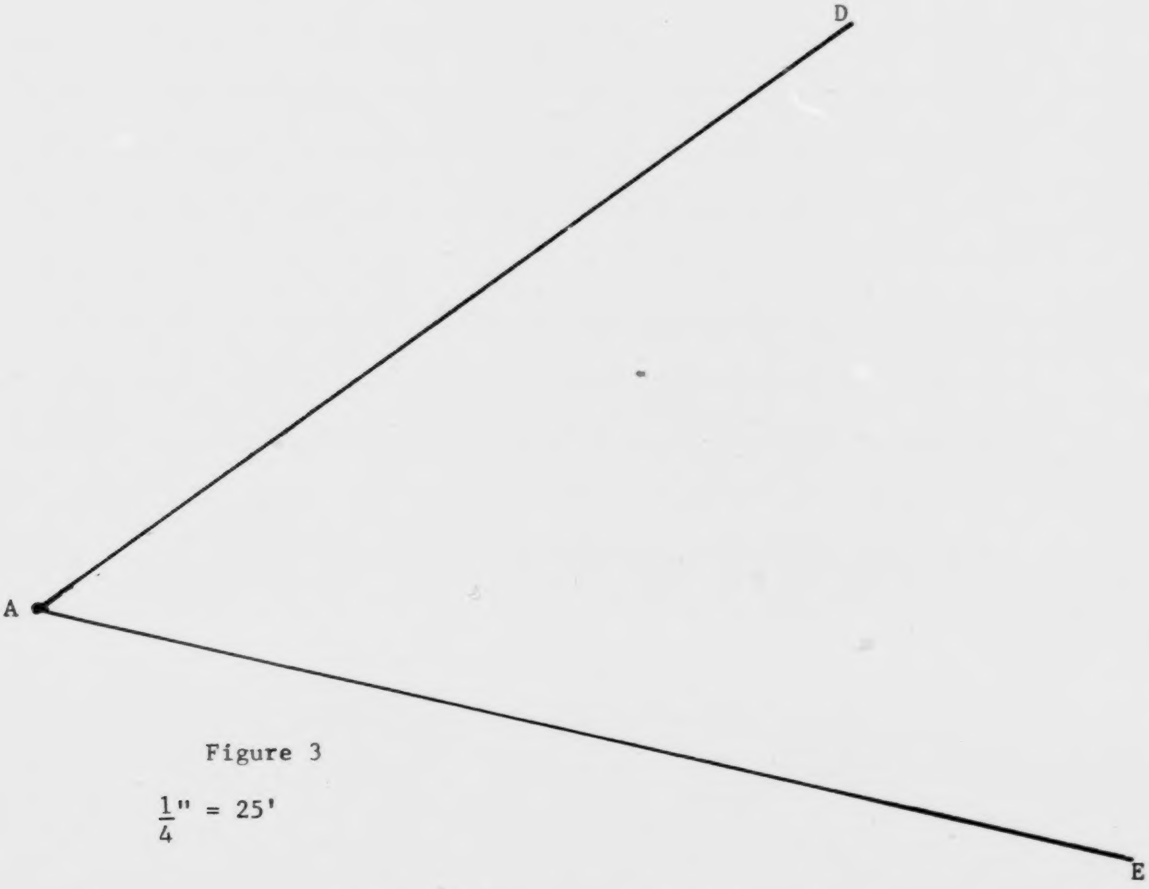


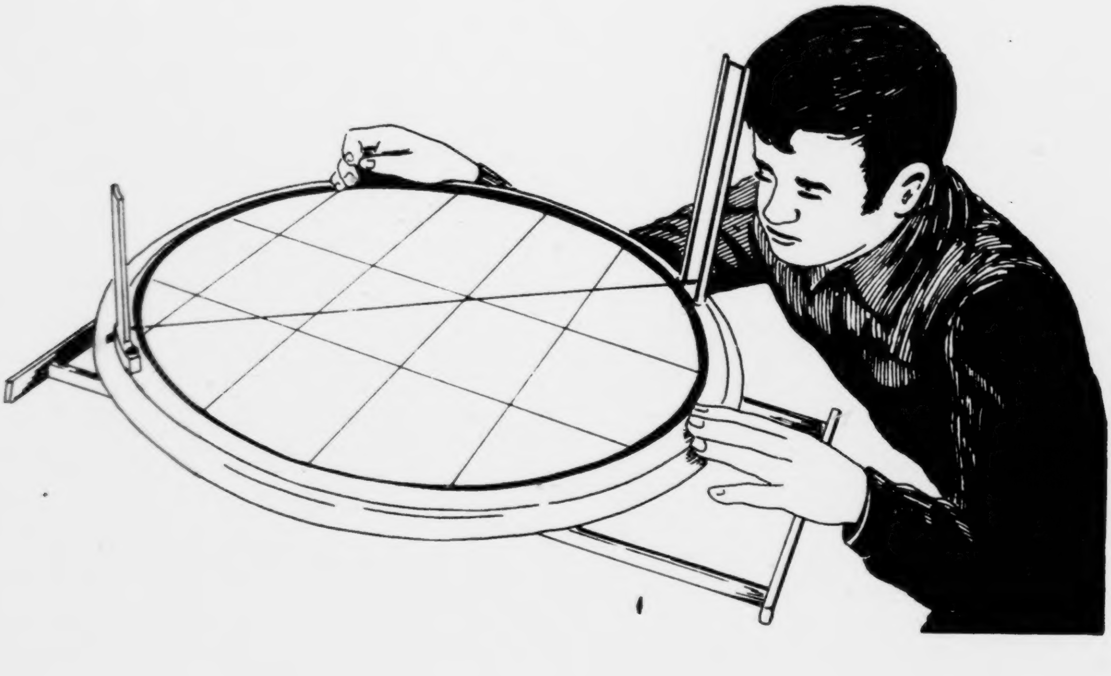
Figure 3

$\frac{1}{4}$" = 25'

FORESTRY

Draw the line B'C' as shown on the sketch of the plane table in figure 1. Since the length of B'C' in the plane-table drawing is the scale length which represents the actual distance from B to C, the length of BC can be determined. What is the length of BC?____________________________

The following picture shows a forest ranger using a plane table to determine the location of a fire.



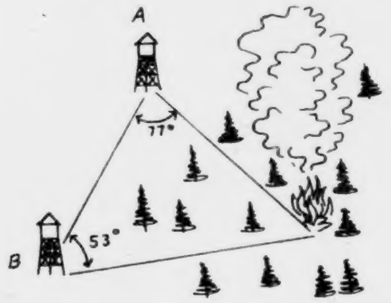
FORESTRY

2. Two forest lookout stations, six miles apart, are at points A and B as shown below. Forest service personnel in each tower observe a fire located at point C. The observer at lookout A finds that the measure of the angle from the fire to lookout B is 77 degrees. The observer at B measures an angle of 53 degrees from the fire to lookout A. How far is the fire from lookout A to the nearest tenth of a mile? a.________________

Hint: one way to solve the problem is to construct a scale drawing and measure the distance. Use your ruler and protractor and construct a scale drawing.

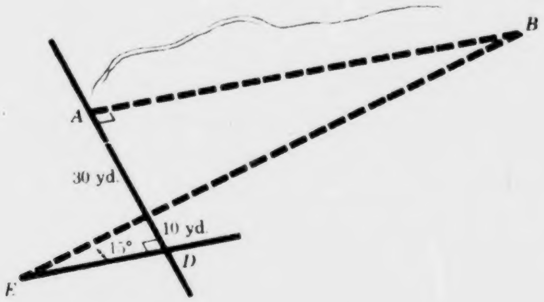
How far is the fire from lookout B? b.____________________

Let your scale be 1/16" = .1 mile



FORESTRY

3. Two camps were located at points A and B on opposite sides of a lake as shown below. Some measurements are indicated on the figure. (m∠BAD = m∠ADE)



a. What additional measurement must be made in order to determine the distance, AB, across the lake? ______________________________

b. Make a scale drawing and find the distance AB to the nearest yard.

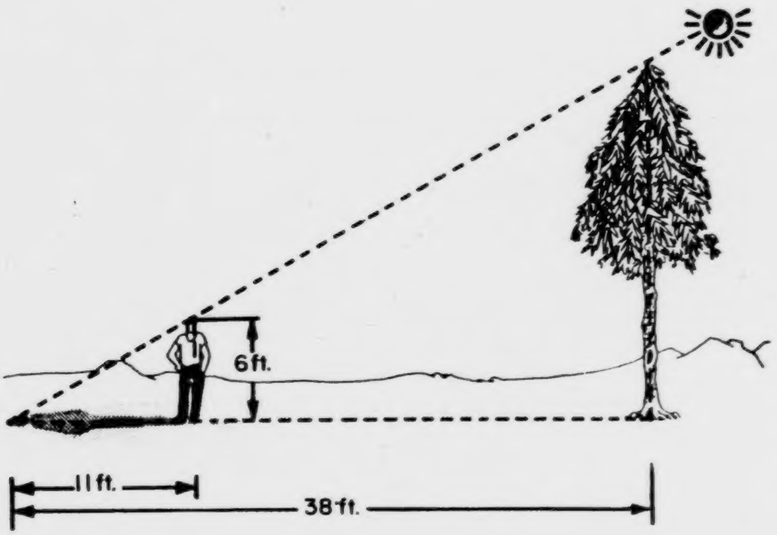
AB = ____________________ Let 1/16" = 1 yard.

FORESTRY

There are times when a forester may have to use indirect measurement, to find the height of a tree for example. The next problem would be one method he could try.

4. How high is the tree if it casts a shadow of 38' when a man 6' tall casts a shadow of 11'? Height = ____________________

   (Use proportions from similar triangles.)

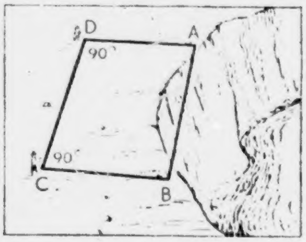


5. A ranger measured the circumference of a tree at waist height. If the circumference is $169\frac{1}{2}$ in., what is the diameter of the tree?

   Diameter = ________________________

FORESTRY

6. The distance between two points on a cliff is to be measured. The rectangle method" is used as shown in the figure.



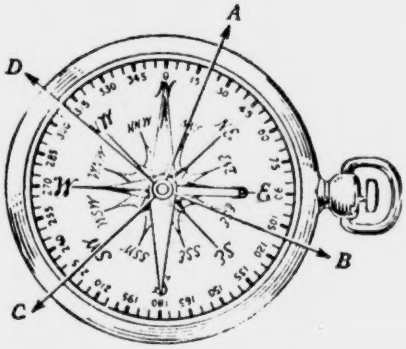
a. What kind of angles are formed at A and B? ____________________

b. What is true about the distances AD and BC? ___________________

c. What distance should be measured to determine indirectly the distance AB? ____________________________

The compass has aided foresters in finding their way in the woods for many years. When in proper operation, the needle turns until it points to the magnetic North Pole. A forester would revolve the compass until the zero on its scale is placed directly at the "north" end of the needle as shown in the diagram.

Now by sighting across the center of the instrument to some distant object and noting where the line of sight crosses the scale, the forester can read the magnetic "bearing," or direction angle, of the object from the position he is in. The observed bearing of A is 20°.



7. What is the observed bearing of

   a. B? ____________________

   b. C? ____________________

   c. D? ____________________

   Notice that with this kind of a compass, all bearings are read "clockwise" from the north.

For further information write:

American Forestry Association
919 Seventeenth Street N.W.
Washington, DC 20006

Society of American Foresters
1010 Sixteenth Street N.W.
Washington, DC 20036

II-15 Electrician

ELECTRICIAN

Jake's folks decided to build an addition on to the house. They had always thought that a recreation room and extra bedroom would be nice. Jake and his dad had done some of the work, but now it was time to put in the wiring and Jake's dad didn't want to tackle that himself. An electrician was hired and Jake got to know him quite well while he was watching hime wire the new addition.

Much of the time they talked about the electrician's work. Jake discovered that the electrician did most of the work the way he did because of codes. These are rules that the state and local governments have adopted to insure safe, attractive, and uniform electrical installations. The outlets that were being installed, for instance, had to be properly grounded.

The electrician told Jake that he was self-employed but that most of the electricians he knew worked for a contractor. He was therefore a little more "on his own," but at times had more trouble finding work because he didn't have the large construction jobs the bigger contractor would get.

Jake asked his dad how much the electrician was going to charge. Jake's dad said that would depend on how long the job took and that all of this talking to the electrician didn't help any. The company was charging $18 an hour plus all the supplies, of course. Jake figured that a couple of days' work by the electrician could really add up. He discovered, however, that $18 an hour was about the average of what was charged in only that part of the country.

Jake had two more questions for the electrician. The first concerned the future of this line of work and the second dealt with the theory of electricity. Jake was told that the electrician of the future could be doing quite a different job. Already a couple of the electrician's friends had gone to work for a firm that made modular or packaged homes. This meant that the work could be done in a "factory style" setting with the wiring and electrical work installed in almost an assembly line fashion. The electrician said this meant faster work under better, more uniform conditions but would be even less appealing to him than working for a large contractor. Jake could understand that this electrician wanted to be more on his own, with the freedom that being your own boss provides.

The electrician then explained some of the theory of electricity to Jake. "Water is measured in gallons, meat in pounds. Electrical power cannot be weighed on a scale because it is in constant motion. Instead, electricity is measured by how much flows past a point in the electric line at any given moment.

"When we want to measure a quantity of water, we talk of 'gallons.' When we talk of water in motion, we talk about 'gallons per second.' With respect to electricity, a unit of electricity in motion is called an ampere.

"When air is put under pressure, we speak of pounds of pressure per square inch. Electric power is also under pressure. This pressure is measured in volts. A small radio battery has two volts of pressure, regular house current has 115 volts of electrical pressure. Do you know how many volts a car battery has? Amperes or volts alone do not determine the amount of power flowing in a wire, the two combined do. Watts is the term used to determine the amount of power (amperes and volts) running through a line. 'Watts = Volts x Amperes.'"

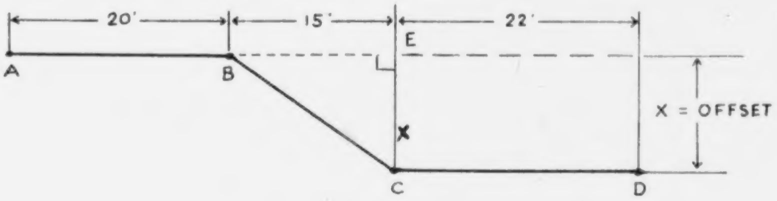
The electrician used the example of a rifle. If the speed of a bullet remains constant (volts) but the weight of the bullet varies (amperes), the impact varies. The opposite is also true. If the weight stays constant (amperes) and the speed changes (volts), the impact varies.

ELECTRICIAN

Much of the mathematics an electrician uses is applied higher algebra and trionometry. He must be able to read blueprints, much of which are in the form of geometric figures. Applied geometry, especially working with angles and missing parts of geometric figures, is a must.

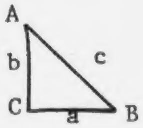
An example of the applied use of the Pythagorean Theorem is shown in the following problem.

1. A 60' length of conduit is bent as shown. Find the distance of X.



Notice Δ BEC is a right Δ. The length BE is known, BC can be found from the diagram (AB+BC+CD=60), and from this X (same as EC) can be calculated.

[Pythagorean Theorem



; $C^2 = a^2 + b^2$ Where C is the hypoteneuse (side opposite the 90° angle) and a and b are the other sides]

Find X: ____________________

Another application of the Pythagorean Theorem is in finding the voltage across two vectors.

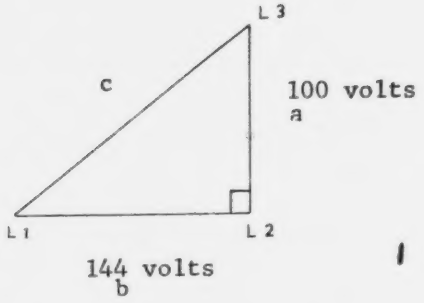
2. Given: a 100-volt line between

   L_3 and L_2

   a 144 volt line between

   L_2 and L_1

   Find: the voltage between L_3 and L_1



Notice that the line between $\angle_3$ and $\angle_1$ would be the hypoteneuse of a right triangle. Thus you can use the formula $C^2 = a^2 + b^2$.

Find C (approximate).

ELECTRICIAN

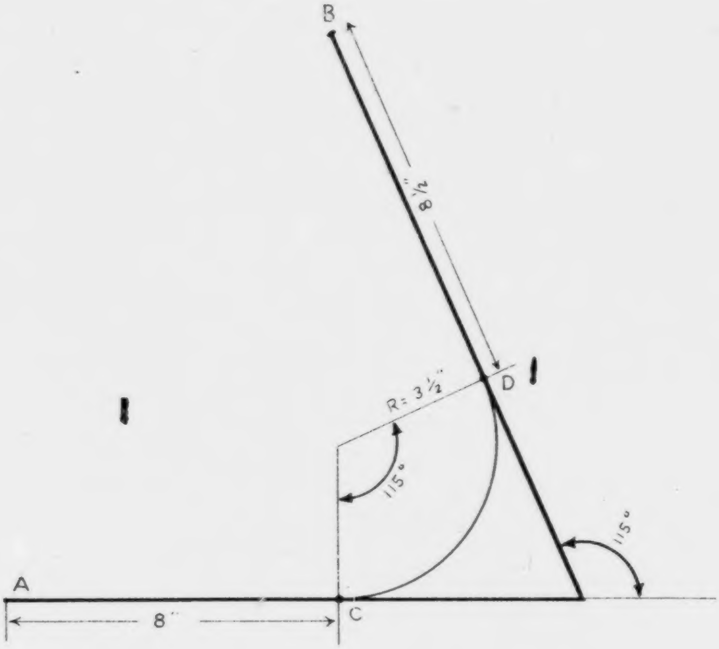
An electrician must also work with circle measurements. The following diagram shows an applied situation. The diagram is of the inside of a roof meeting an inside ceiling. He must figure what length of pipe to use, including the bend, to go from A to B. The electrician has found the center of the circle whose arc is DC and radius is 3½".

Find the total circumference of the circle O, then find the length of arc $\overset{\frown}{DC}$. Let $\pi = 3.14$.

3. a. Circumference of circle O = π x D = __________

   b. Length of arc $\overset{\frown}{DC} = \frac{115^\circ}{360^\circ}$ of C = __________

   c. Total length of conduit needed to go from A to B = ________________



ELECTRICIAN

The electrician must also be a good carpenter. His knowledge of basic geometry is essential to his everyday work.

II-16 General Contractor

GENERAL CONTRACTOR

Sam wanted to work in many construction fields. He didn't want to limit himself to just one, such as carpentry, plumbing, or electronics. Yet he knew that it would be difficult to become skilled in all the trades, not to mention the tremendous cost of being equipped in so many occupations.

So he decided that maybe a better solution was to become a general contractor. A general contractor takes responsibility for the overall construction of a building. Much of the work he is directly responsible for. Some phases of the work, such as electrical work or plumbing, he contracts other tradesmem to do.

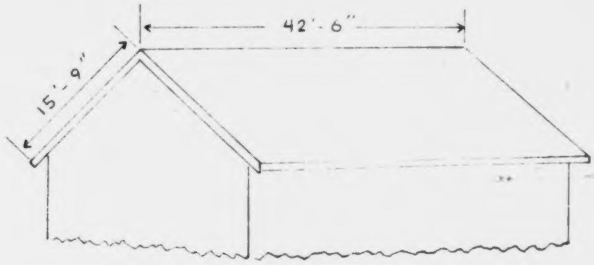
While looking through the math he should be responsible for knowing, he found that he must know how to make accurate estimates of a job. He must have enough knowledge of other trades to be able to give good bids for a job. He then contracts other tradesmen to do various phases of construction that need to be done and which he is not equipped to do. Here are some examples of problems a general contractor might have to do.

1. The basement of a house 28' long and 24' wide is to be excavated to a depth of 6'. A building ordinance requires that the excavation be extended 1' beyond the wall on each side to permit inspection of the masonry. At $1.25 per cubic yards, find the cost contracting an excavator. Cost = ________
(Volume LxWxH)

2. The house in problem one is located on a lot 120' long and 45' wide. The level of the lot is to be raised 18". Find the cost of filling at $.90 per cubic yards.
Cost = ____________________

GENERAL CONTRACTOR

3. Both sides of the roof shown are to be covered with asphalt shingles. Find the number of sq. ft. of surface to be covered.



Number of sq. ft. = a.__________________

If 100 sq. ft. = 1 sq. of shingles and if 1 sq. of shingles comes in 3 bundles, then how many bundles must be ordered? b. __________________

Adding one-third square for a Boston Ridge, how many bundles must be ordered in all to complete the job? c.__________________

A contractor can sub-contract a shingler to put on the roofing for $5 a square. What will the total cost be? d.__________________

4. A contractor must estimate the excavation and rock fill needed for two circular dry wells. How many cubic yards of rock fill will it take if one well has a diameter of 5'-0" and a depth of 6'-6" and the other has a diameter of 5'-6" and a depth of 7'-0"? (Vol. of cylinder = πr^2 x H)

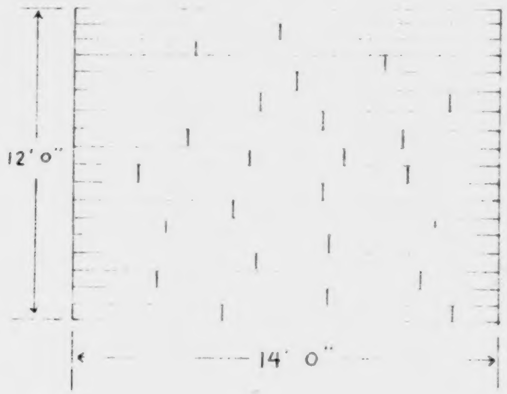
Cu. yd. = __________________

5. For estimating purposes, when computing labor costs for finish flooring, the actual area of the floor, in sq. ft., is used. An average carpenter can lay, scrape the joints, and sand approximately two squares (200 sq. ft.) of finish flooring in an eight-hour day.

How long will it take to lay finish flooring in the room shown below?

Hrs. = ________________ At $6.80 per hour, what is the labor cost?

Cost = ________________



6. Read the blueprint on the next page.

a. What is the length of the house measured along the west wall?

Length = ________________

b. What is the perimeter of the house?

Perimeter = ____________

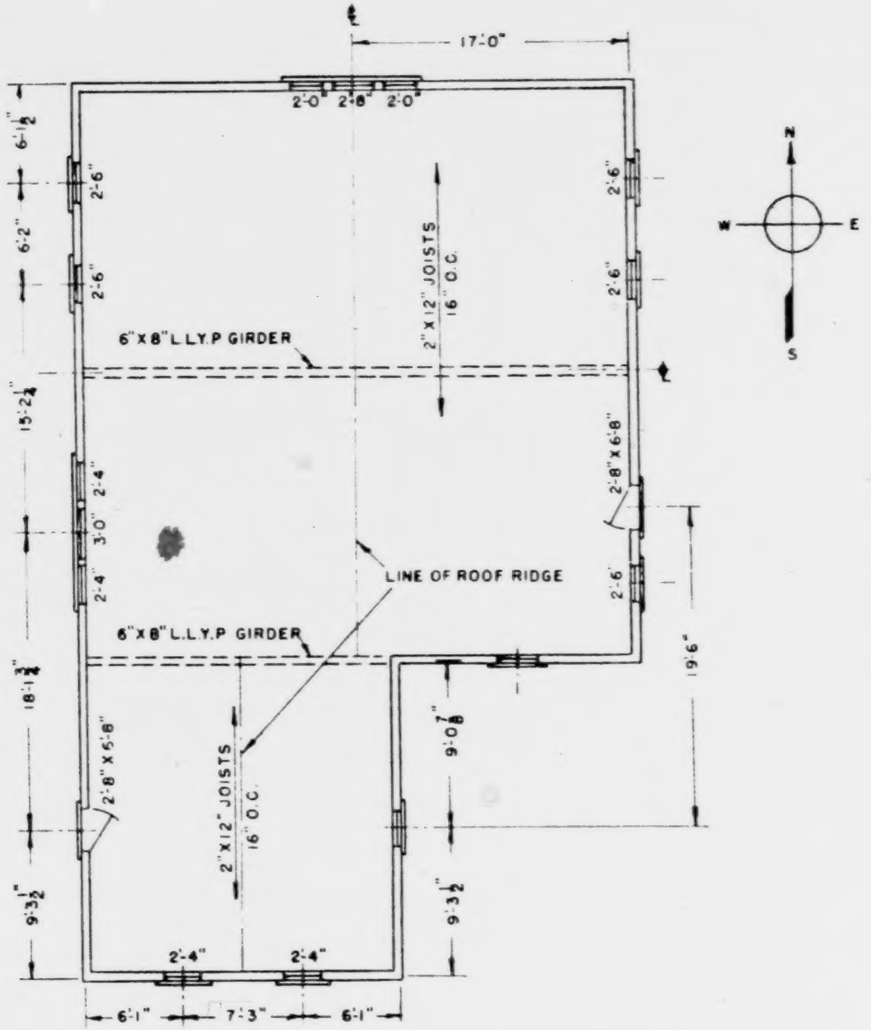
c. How many joists will be required for the south span?

Number of joists = ________________

d. At $6.04 each, how much will the joists for the south span cost?

Cost = ________________________

GENERAL CONTRACTOR



Sometimes it is necessary for you to find the area of a triangle without knowing the height. If you know the lengths of the three sides are a, b, and c, then use the following formula:

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where} \quad s = \tfrac{1}{2} \cdot (a+b+c)$$

7. Some triangular-shaped land was offered for sale at $3,000 per acre. A contractor was interested in buying the land to plot into smaller lots and build homes on them to sell. If the sides of the land were 50 rods, 90 rods, and 100 rods, find the area of the land in sq. rods.

Area = ____________________

Since 160 sq. rods = 1 acre, what is the sale price of the land?

Price = __________________

8. Concrete is obtained by mixing 4 parts gravel, 3 parts sand, and 1 part cement, by volume. Determine the number of cubic yards of each material required to make 150 cubic yards of concrete.

Gravel = ________________

Sand = _________________

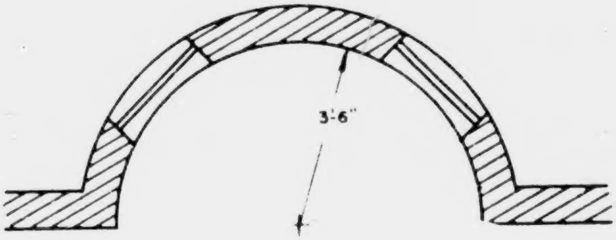
Cement = _______________

9. A porch roof is to be supported by two solid concrete pillars. The pillars are each 2'-0" in diameter and 10'-0" high. How many cubic feet of concrete will the two pillars contain? ____________________ ($V = \pi \times r^2 \times H$)

At $21 per yard (cu. yd.), how much will this cost?

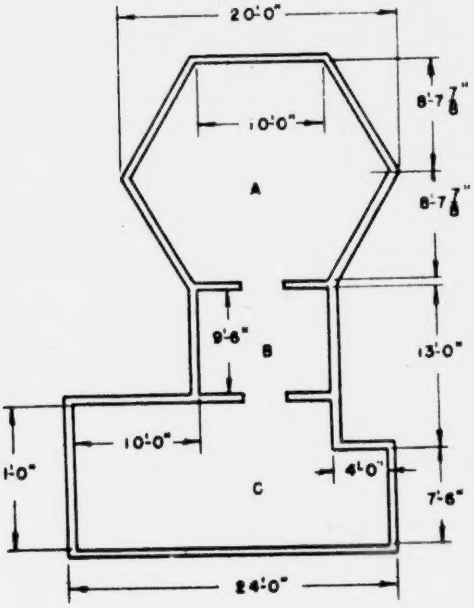
Cost = _______________________

10. Compute the surface area to be floored in the semi-circular bay window shown below. ____________________



11. How many sq. ft. of subflooring are required to cover all the floors of the rooms below?

Area = ________________________



II-17 Home Planning

HOME PLANNING

"Home's not merely four square walls -- home is where affection calls." "What's that got to do with all this?" Tom asked Judy. They were trying to design the living arrangements for their new home and Tom was determined to slap up four walls and be done with it. It would be much cheaper and they could make use of their present furniture, rather than buy new units for everything. He was exasperated with Judy's constantly changing ideas on arrangement. Tom preferred having the fixed items built-in since that would give a neater arrangement as well as have the main part of the floor in any room free for any group type activities they might wish to plan.

The first thing they had done was contact an architect for the basic layout and plans. Then they could decide whether they'd do the work themselves or contract it out to one of the many registered remodelers available.

First Tom and Judy tackled the kitchen. Many basic arrangements were available depending upon what their home and entertainment ideas were, from the one-wall arrangement common in apartment buildings and small homes to the most popular U-shaped kitchen. In all cases, though, they had to be conscious of the work triangle connecting the range, refrigerator, and sink centers. For maximum efficiency, the triangle perimeter should not be more than 22 feet. Doors should be located so that traffic lanes do not cross the work triangle. It is generally accepted that distances between appliances should be four to seven feet between refrigerator and sink, four to six feet between sink and range, and four to nine feet between range and refrigerator.

HOME PLANNING

Below you will see some of the arrangements available to Tom and Judy. In each case, you are to:

A. Draw the step-saving triangle. (Use the points given for accuracy.)

B. Label the sides a,b,c.

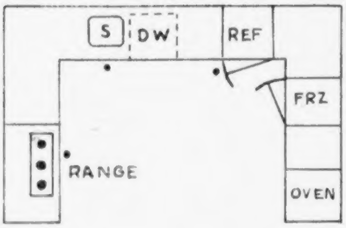
C. Measure a, b, c (scale $\frac{1}{8}$" = 1')

D. Compute the perimeter: $P = a+b+c$

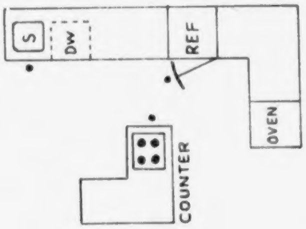
E. Compute the area by using Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{1}{2}(a+b+c)$

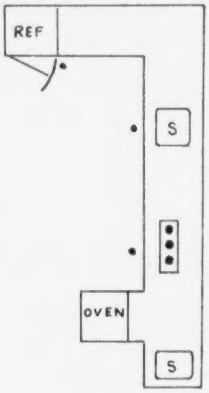
I



II



III



1. U-shaped kitchen

A. Draw

B. Label sides

C. a = ____ b = _____ c = _____

D. P = _________

E. S = ________; A = _______

2. Island Arrangement

A. Draw

B. Label sides

C. a =_____ b = _____ c = _____

D. P = _________

E. S = _________;A = ________

3. Peninsula Arrangement

A. Draw

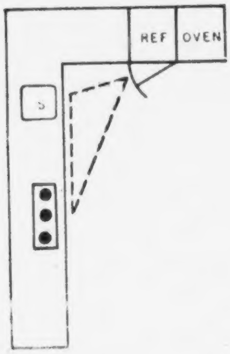
B. Label sides

C. a=_____b=_____c= ___

D. P = ________

E. S = _____; A = _____

IV

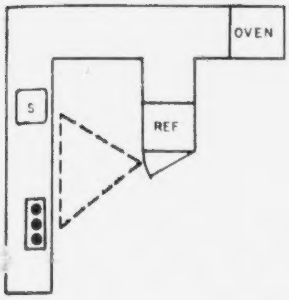


4. L-shaped

A. P = ____________

B. A = ____________

V

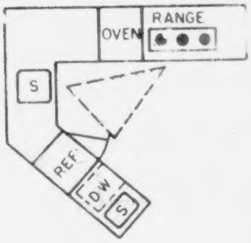


5. Peninsular

A. P = ____________

B. A = ____________

VI



6. Modified U-shaped

A. P = ____________

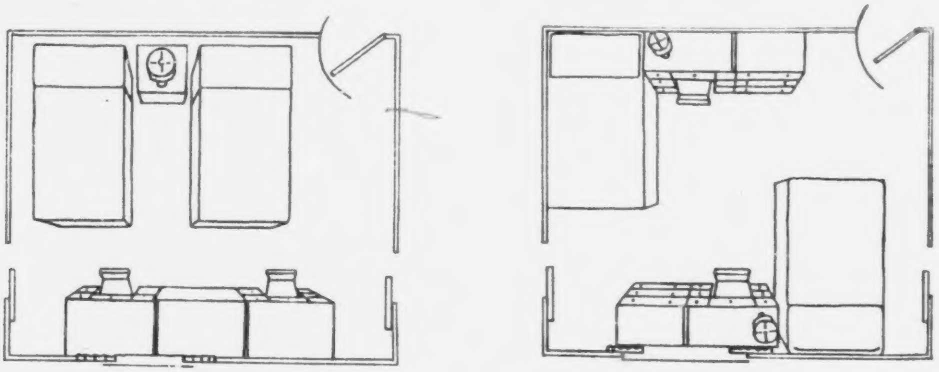
B. A = ____________

Tom and Judy felt the organization of the kitchen was most important since it would be used in most activities in the house for both family living and entertaining. They could not afford a second kitchen in the recreation room, so their main kitchen would serve double duty. Really, this was Judy's opinion and Tom thought best to agree with her, especially since he considered her better skilled at home planning than he.

The layout of the entire home was tackled room by room and many different decisions had to be made. Furniture arrangement had a greater importance than Tom first realized. Until the architect explained flow patterns to him, he had considered Judy's occasional furniture re-arrangement sprees merely exasperating feminine whims.

HOME PLANNING

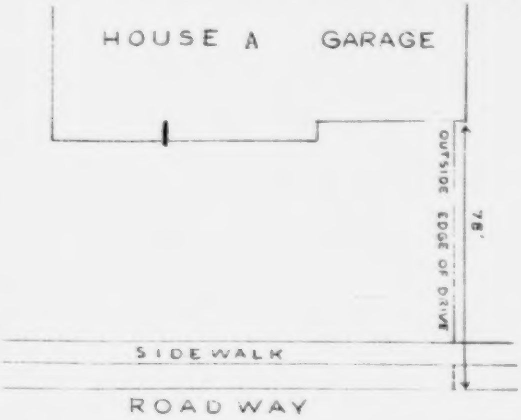
To illustrate without getting into lengthy discussions, consider the sleeping arrangements for Tom and Judy's two girls. They had to share a bedroom so one room would have to contain two single beds as well as a small chest of drawers and a vanity for each girl. Needless to say, Tom and Judy would also like to keep as much available floor space for the girls to use. A glance at the two arrangements shown should clarify the points being made.

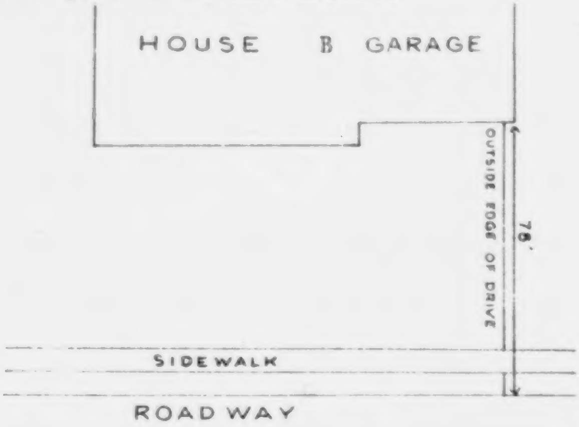


The usual side-by-side arrangement of twin beds leaves little open floor space. In many rooms, the beds can be separated to leave more open floor space, and each person has his own area.

Another problem that Tom and Judy had was that their house was located on a rather busy street. It would be desirable not to have to back out onto the street where there would always be a possibility of accidents. Two basic plans seemed to suit their purposes. The first was a circular drive which might be added to the straight drive heading to the garage for a turnaround; the second would be a spur added to a straight driveway; this would provide additional off-street parking or space for turning around in order to head into street traffic.

On page five are sketches of the two possibilities and you are to construct accurate layouts of the circular drive and the drive with the spur addition. The measurements are all given below and you are to use a scale of 1/8" = 6' for your construction. In some cases, six feet does not divide evenly into the dimensions needed, so you will need to first calculate the fractional part of an inch needed.





Specifications — Scale Dimensions (Answer in inches)

Driveway: 18' wide = Example 7. $\frac{3}{8}$"

Length of main drive: 78' = 8.______

Circular drive:

Inside radius: 18' = 9.______

Width of drive: 12' = 10.______

Outside radius: 18' + 12' = 11.______

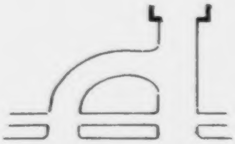
*The circles are concentric and their center lies on the house side of the sidewalk; also, the inside circle should be tangent to the main driveway.

Exit of circular drive:

Width: 12' = 12.______

Length: 15' = 13.______

General appearance:



Specifications — Scale Dimensions (Answer in inches)

Driveway: 18' wide = 14.______

Length of main drive: 78' = 15.______

Spur:

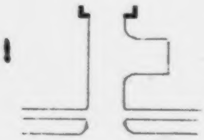
Radius of curved portion: 9' = 16.______

Length of non-curved portion: 9" = 17.______

Width of non-curved portion: 12' = 18.______

*Curved portions of spur should be tangent to both main drive and non-curved portions of the spur.

General appearance:



HOME PLANNING

Tom and Judy wondered about the relative costs of paving both driveways. In order to determine that, they would have to know the total area of the drives and then, assuming a thickness of 4" of concrete, find the volume of concrete needed in cubic yards. In order to see what calulations would be involved, determine the surface area of the circular arrangement only. For our purposes, don't worry about the part of the circular drive that overlaps with the main drive. The extra concrete ordered because of this will be used for fill in back of the house. Use the following outline to aid your thinking:

19. Length of main drive:____________________
20. Width of main drive:____________________
21. Area of main drive:____________________
22. Radius of larger circle:________________
23. Area of larger semicircle:______________
24. Radius of smaller circle:_______________
25. Area of smaller semicircle:_____________
26. Area of circular drive:_________________
27. Total area of drive:____________________
28. Volume of drive (in cubic feet):__________
29. Volume of drive (in cubic yards):_________
30. At $21 per cubic yard, cost of paving drive:____________________

HOME PLANNING

Tom and Judy also calculated the volume of the drive with the spur to compare the cost. By calculations similar to the one outlined above, they learned that driveway B would require $20\frac{1}{2}$ cubic yards. At $21 per cubic yard, that layout would cost 31.______________. They realized that money would not be the only consideration in choosing which drive to install. Which one would you choose, both for convenience as well as cost? 32.___________________________________

____

As is easily seen, all these improvements would be costly, but Tom and Judy thoroughly enjoyed planning the home. They realized that working with an architect and a trained home economist was crucial for the end result to be both efficient and enjoyable. Only part of the planning which goes into a home could be illustrated here, but it is hoped that enough was given so that you may also have some ideas when, in later years, you plan your home.

II-18 Cabinetmaking

CABINETMAKING

After Bill finished exploring the use of geometry as a carpenter, he got to thinking that the math involved wasn't all that bad. The thing that bothered him was working under outdoor conditions. When the weather is nice it's great, but you can't always depend on nice weather. When you consider winter, rainy, and windy days, there might be some miserable days to work in and possibly even weather lay-offs.

Bill did like woodworking, though, so looked into cabinetmaking for a career. The work requires creativity and precision with hand tools. Besides, it would be inside work.

Bill found much similarity between the jobs of a carpenter and a cabinetmaker. In fact, many cabinetmakers start out as carpenters and many others work in both fields, depending upon the needs at the moment. The work seemed to require a little more precision and almost the whole story of a finished product seemed to be found in the way the cabinet looked when finished. Bill liked that, for making "something just perfect" was fun. He found the pay to be much the same for both trades and most of the other factors he looked into overlapped between the two.

A big question for Bill was, "Do I have what it takes to be a cabinetmaker?" For help in answering this question, he went to see his counselor. The counselor looked at Bill's grades and found that he had done quite well in shop courses. In addition, some tests showed that Bill had much better than average aptitude in mechanical and spatial areas. The counselor asked Bill about his hobbies and found that he liked to do creative things, especially those which involved working with his hands. It seemed to the counselor that when all of these things were put together, Bill had the ability and the interest to do the work required of a cabinetmaker.

A second question was, 'Would I have more difficulty with mathematics as a cabinetmaker? Bill decided to find out what types of problems he would encounter while making cabinets or furniture. Here are some examples of math involved.

1. Suppose you see a picture or a sketch of a furniture piece you would like to make. In many cases the three major dimensions of height, depth, and width are given. With careful planning you can design a product that is very similar. Consider the following diagram on the next page and follow the directions using figure 3.
   a. Fasten the picture or sketch to the left edge of a piece of paper.
   b. Use a T-square to extend the straight lines from the edges of the furniture piece.
   c. Suppose that the overall height is 30". Select a scale in which 30 units equal in length will fit diagonally between the two horizonal lines. Place the first division point on the scale on the top line and the last on the bottom line.
   d. Now extend the lines (in b) until they intersect the diagonal line for each dimension needed. Now you can count the number of units or inches for this part. The same method can be used to find other dimensions.
   e. Find the following dimensions: (See diagram on following page.)

A = ____________	E = ____________
B = ____________	F = ____________
C = ____________	G = ____________
D = ____________	H = ____________

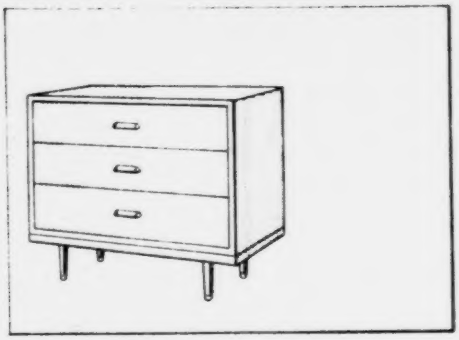


FIG. 1

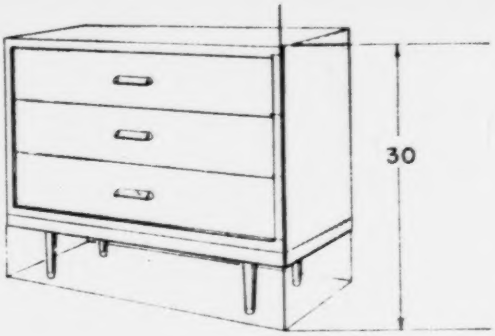


FIG. 2

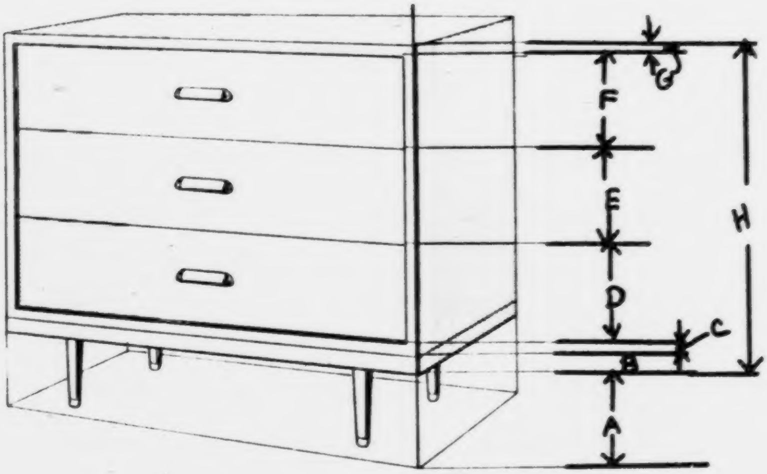
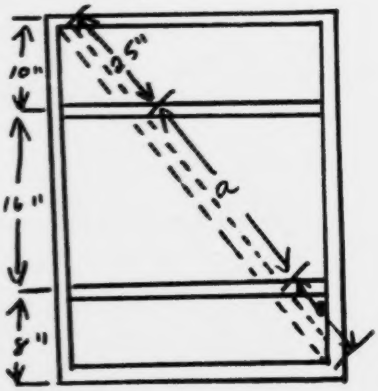


FIG. 3

Determining the dimension of the parts of this chest. The overall size is 30"W x 15"D x 30"H.

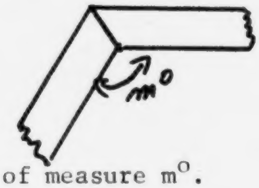
2. A bookcase is arranged so that the spaces between its shelves are 10", 16", and 8" respectively. If a slanting board is nailed diagonally across the back for re-enforcement and cuts off a segment of 25" between the first two shelves, what segments are cut off between the remaining shelves? (Hint: Use ratios; see "Parallel Lines" unit.)

a.________________ and b.________________



CABINETMAKING

3. An octagon is often used for wood products such as wastepaper baskets and small tables. What would be the angle of the cut for the edge of each face of an octagon end table if you make a mitered cut? (Assume the octagon is regular.) Hint: find the measure m° (See unit on polygons.)



and take $\frac{1}{2}$ of measure m°.

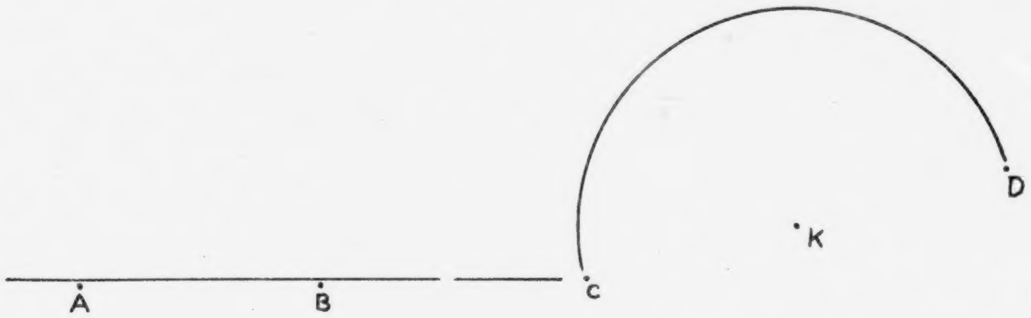
$\frac{1}{2}$ m° = ____________________

4. Many irregular shaped objects such as Early American furniture pieces have arcs, circles, and lines which are tangent. Arcs or circles are tangent when they touch at only one point but do not intersect.

   a. Given the radii of 3 arcs as 2 in., 1 in., and ½ in., construct a series of three tangent arcs. Use given point O as the center of the arc with a $\frac{1}{2}$ in. radius so that the three arcs are tangent in a series.

Make all the construction marks light and show the arcs in bold lines.

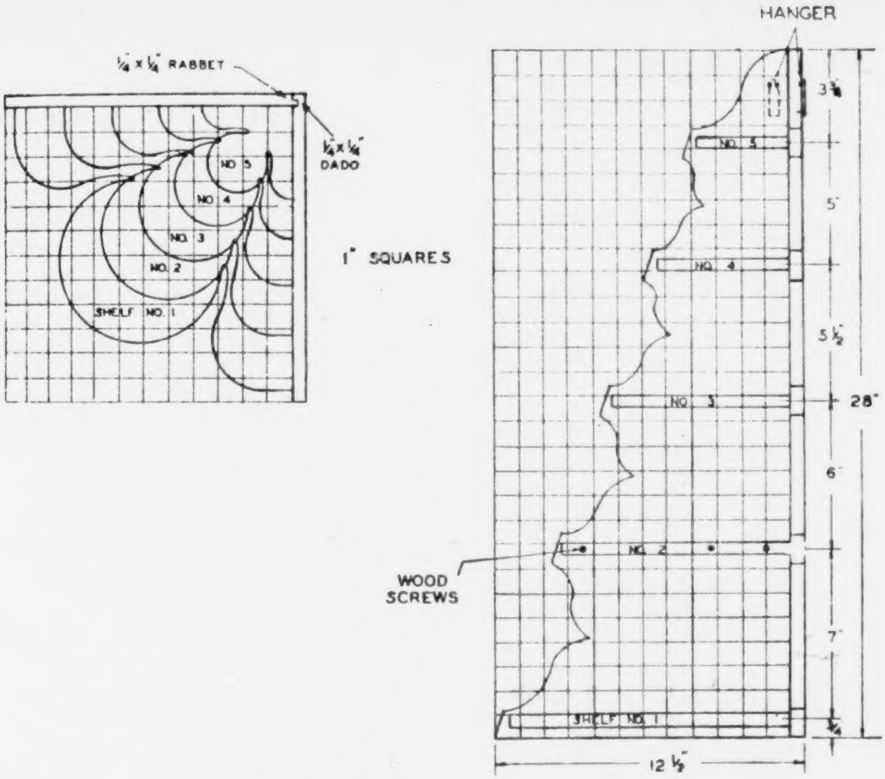
b. Draw an arc with a radius of 1 inch tangent to the given line $\overline{AB}$ and the given arc $\overset{\frown}{CD}$. The center of arc $\overset{\frown}{CD}$ is K.



5. What length of 1" veneer strip is needed to go around a round table with a diameter of 54 inches? ____________________
Use π = 3.14.

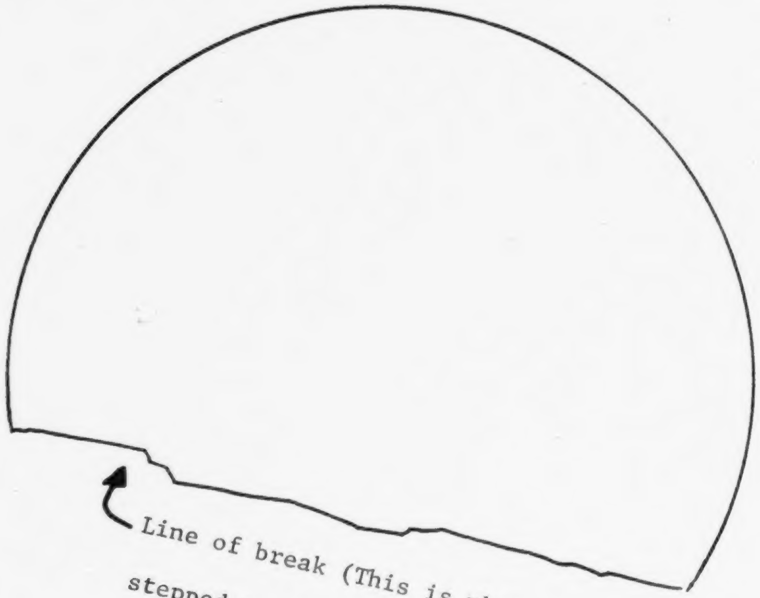
6. If you used a 2'-0" square piece of plywood to make a 2'-0" diameter round table top, how much wood is wasted?
Sq. ft. wasted = ____________________ (See unit on polygons and areas.)

7. Projects found in books and magazines are seldom drawn to full size. If the product contains irregular parts, it is necessary first to enlarge these to full size or scale. In the scale drawing, on the following page, of a corner shelf, consider each square equivalent to a 1" square. Make 1" squares and transfer the design of one shelf or one side to the 1" squares. (Hint: continue to locate and transfer points until enough are marked for a pattern. Sketch the curves lightly freehand, then use a French curve to darken the lines and to produce a smooth, evenly curved line.)



8. On a drawing of a scale of 1/16" = 1'-0", what would be the scaled measurement of a line representing 100'-0" ? ____________________

9. A circular table top is broken and the loose piece has been lost in shipment. You have been asked to duplicate this top. You have an idea where the center of the diameter is. Show on the diagram (on the following page) how you would locate the center to duplicate the table top. (Hint: work with the perpendicular bisectors of a chord.)



Line of break (This is where big Bertha stepped on the table.)

II-19 Plumbing and Pipe Fitting

PLUMBING AND PIPEFITTING

Many people link plumbers and pipefitters together; there are some likenesses, but there are many differences. A plumber installs and maintains piping systems which carry water for domestic and commercial uses. Installation of waste pipes and plumbing fixtures as well as gas lines are his main concerns. A plumber must be able to select pipes, valves, and other fittings appropriate for the job and state and local codes.

A pipefitter-steamfitter works in such new areas as atomic energy power houses and total energy systems as well as the older steam systems of heat and energy. He must apply a technical knowledge of welding, brazing, soldering, bending, and threading in the installation and maintenance of piping systems. He must have a working knowledge of the various types of power plant installation as well as heating systems and process piping systems.

An individual interested in becoming a plumber or pipefitter can go about it in a couple of ways. There are at least four vocational-technical schools in Minnesota where an individual can get training. The course is 10 months long and is available at St. Paul, Jackson, St. Cloud, and Wadena. This schooling will cut down on the length of time that a person must serve as an apprentice.

Others may start out as apprentices without trade school preparation. They begin work at 40 to 50 percent of the journeyman's wage and need to spend five years in the training program. During this time, they will take realted classroom training. There are 350,000 plumbers and pipefitters at the present time and this total is expected to increase as construction activity continues to increase. These tradesmen earn an average of $7 per hour and can earn as much as $9.42 as plumbers in Oakland, California, do. In addition, they usually receive many desirable fringe benefits.

Ability in areas of physics and math are more important than one would think because many applications of the angle, volume, and circle principles are involved.

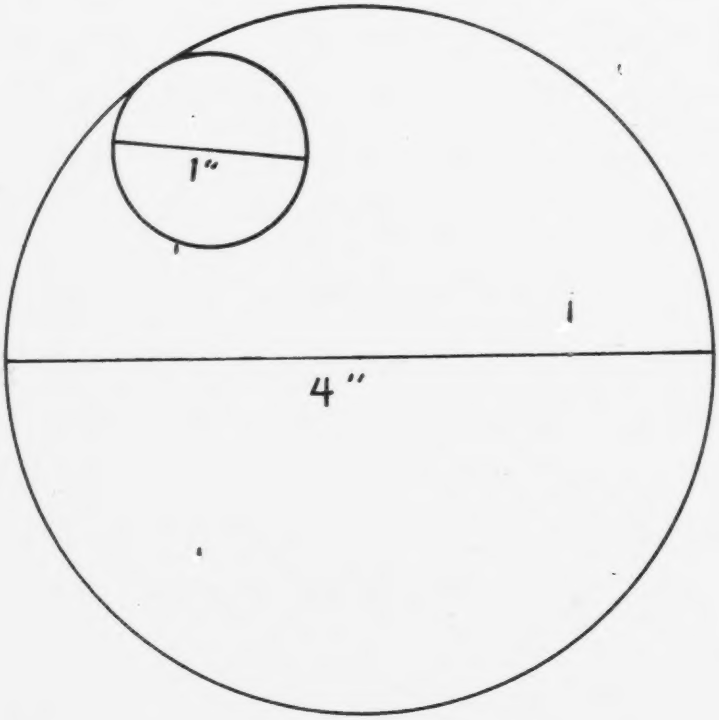
People working on both plumbing and pipefitting must have an understanding of the carrying capacity of different size pipes.

Example: A 4" water pipe can carry how many times as much water as a 1" pipe?

Solution: A quick answer would be four times, but let's take a look at a drawing of the two.

As you can see in the diagram, many more than four 1" pipes are contained in a 4" pipe. To see the solution geometrically, we can compare the areas of both pipes.

End View of Pipes



PLUMBING AND PIPEFITTING

A = π x radius squared, π = 3.14 or $\frac{22}{7}$

$$\frac{\text{area of 4" pipe}}{\text{area of 1" pipe}} = \frac{\cancel{\pi}\,2^2}{\cancel{\pi}\,r^2} = \frac{\pi\,2^2}{\pi\,\frac{1}{2}^2} = \frac{4}{\frac{1}{4}} = \frac{16}{1}$$

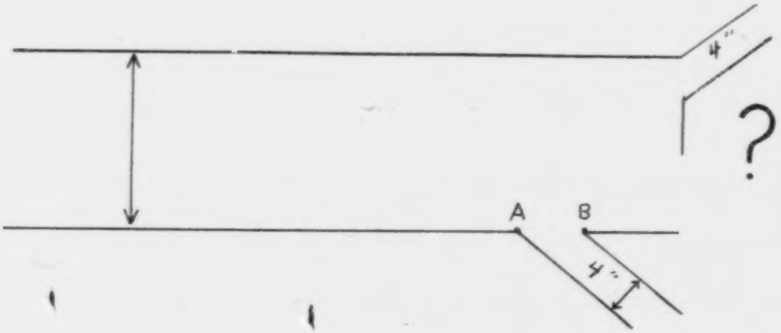
Therefore the 4" pipe carries 16 times more water than the 1" pipe.

1. Find how many times as much water a 6" pipe carries than a 1" pipe.

2. How much more water will a 6" pipe carry than a $1\frac{1}{2}$" pipe?

3. An 18" main steam pipe coming from a boiler can have a number of 4" branch pipes from it to equal the area of the main. How many 4" branch pipes can be installed?

   Hint: Use same idea as in previous example. Determine the area of the main pipe and then the area of one 4" pipe.



PLUMBING AND PIPEFITTING

4. Three separate pipes have been used to carry drainage from the second floor of a house. If their diameters are 2", 3", and $1\frac{1}{2}$", find the diameter of a single pipe having the same carrying capacity. Make a drawing of the single pipe.

Diameter ____________________

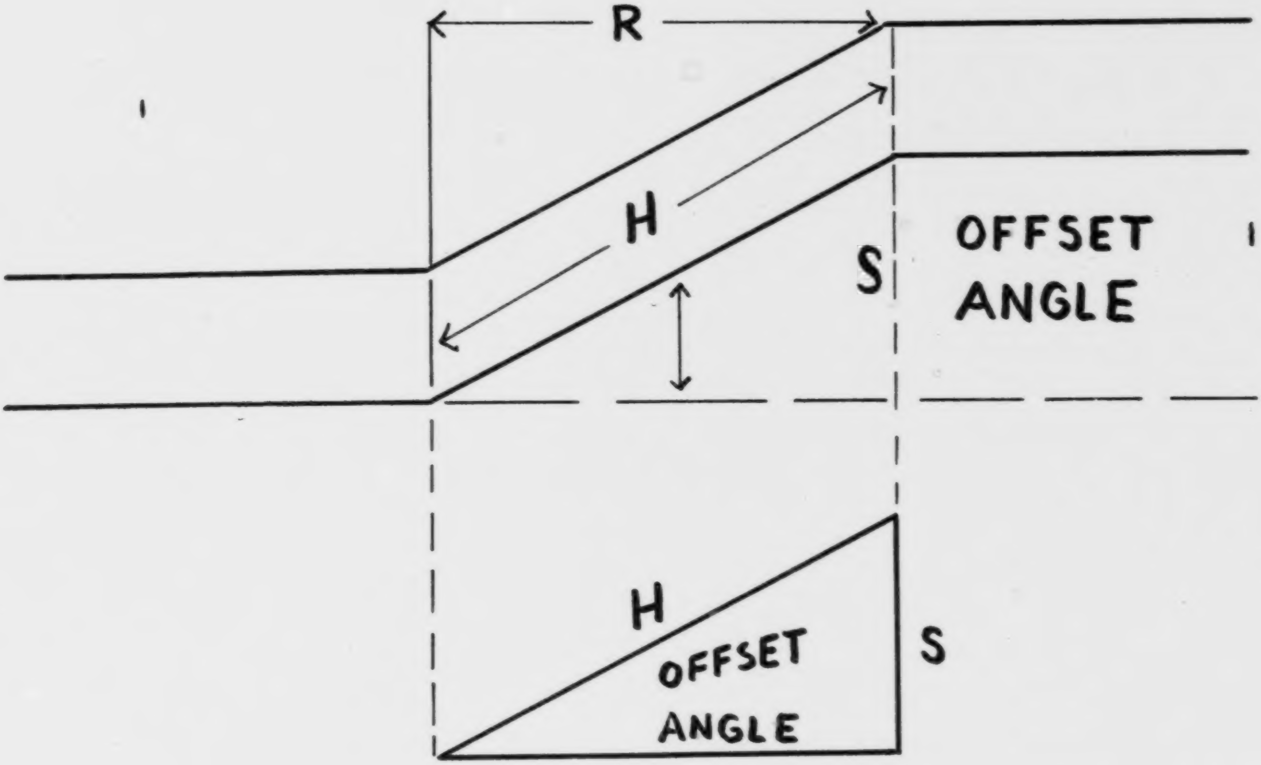
5. By doubling the inside diameter of a pipe, how many times does the capacity increase? Choose at least two examples of pipe to double.

6. The diameter of a piston in a steam pump is 8 inches. When the steam pressure is 80 pounds per square inch (PSI), the total pressure on the piston is? (Determine the number of square inches of the piston then multiply by 80 PSI.)

Pressure = ____________________

Whenever the run of a pipe must change directions, the angle formed by this deviation is known as the offset angle.

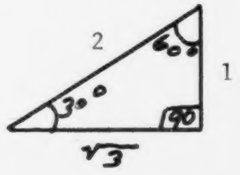
Example



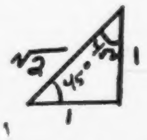
Notice that if we use S, R, and H from the diagram, a right triangle is formed.

Common offset angles are 30°, 45°, and 60°. These situations, then, are examples of the practical applications of a 30°-60°-90° △ and 45°-45°-90° triangles.

Recall: the ratio of the sides of a 30°-60°-90° is

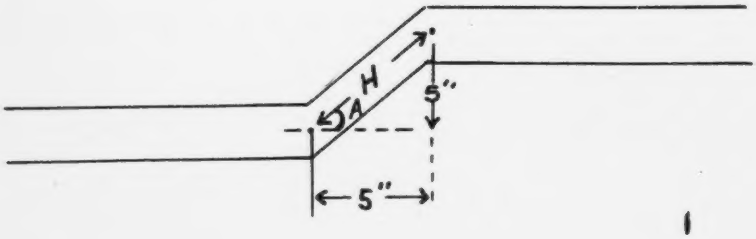


and of a 45°-45°-90° is



.

7. A plumber must adjust his run so that S = 10" and the offset angle is 30°. How long a piece of pipe (length x) must be cut for the distance H? (Refer to the 30°-60°-90° triangle.)



Find the offset angle A if the height is 5" and the run is 5". (See diagram.)

(Refer to the 45°-45°-90° triangle.) m∠A a. ____________

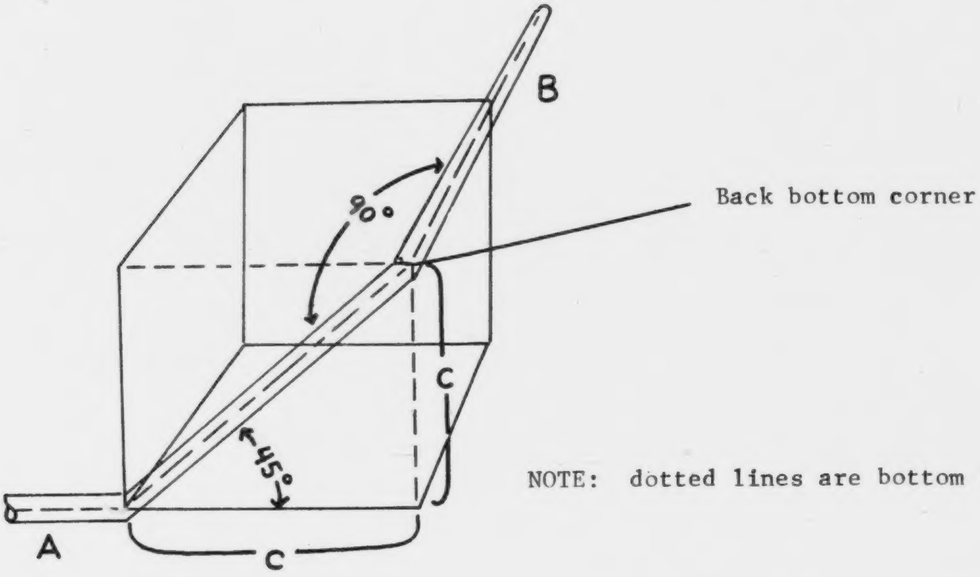
What length pipe is needed for H? b.____________________

PLUMBING AND PIPEFITTING

A more difficult offset is the perpendicular offset. It is one in which the run after the offset is perpendicular to the run before the offset.

Notice that pipe A is "perpendicular" to B (actually, their planes are $\perp$). If the pipefitter knows that the distance C is 36", how long a section of pipe is needed for the diagonal run?

8.______________________



NOTE: dotted lines are bottom lines

PLUMBING AND PIPEFITTING

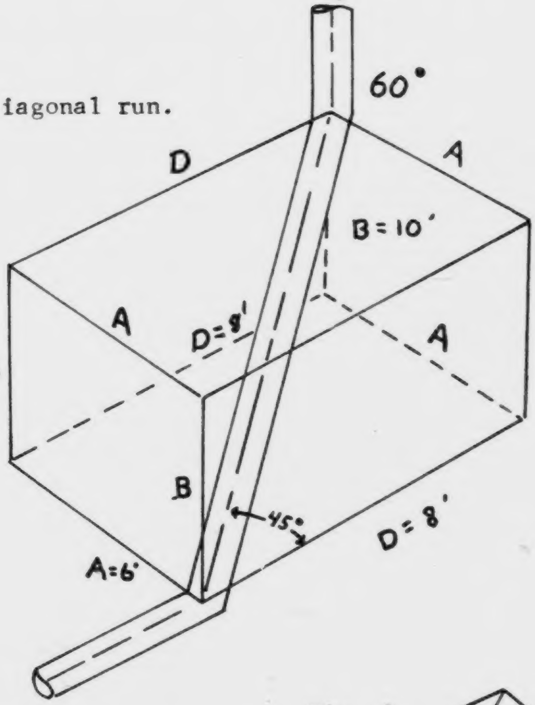
For the job pictured at right, the pipefitter must find the diagonal of a rectangular solid to find the length of the diagonal run.

If A = 6'

D = 8'

B = 10'

Fig. 1

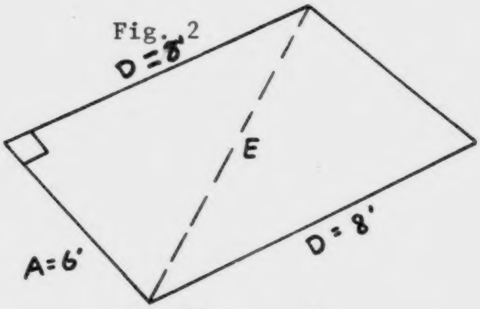


Step 1

Find the length of the diagonal of the bottom rectangle. Use the Pythogoran Theorem $E^2 = A^2+D^2$

9. E =________________________

Fig. 2

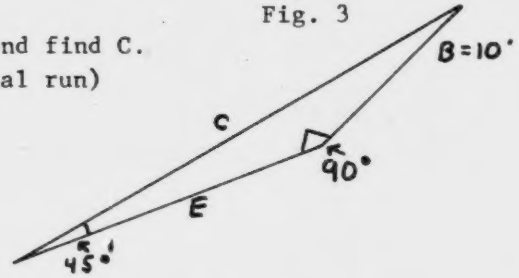


Step 2

Use E from fig. 2 as the side of a triangle and find C.

10. C = ____________________(length of diagonal run)

Fig. 3



The plumber and pipefitter must spend hours of preparation in math. But the learning process is much easier because they are working with material and realize that this will be their occupation. After a few year on the job, they develop "a feel" for their work and can readily cut pipes and choose fittings for the proper installation.

SURVEYOR

One day during lunch, Chuck was talking to his friend about summer jobs. Chuck had been wondering what he was going to do. He hadn't worked previous summers, but now he needed some extra money for his senior year.

His buddy told Chuck to check into getting a job as a rodman with a surveying crew. That was the job he had, and the money wasn't too bad. Chuck asked his friend to tell more, especially how to get a job like that.

He found that a rodman holds the pole, assisting in measuring the height or distance or direction that a surveyor is working. Chuck was told that most jobs like this are with a government agency (county, city, or state) and the surveying crew is "laying out" roads, etc.

Chuck was concerned -- he didn't know much about surveying and would have to learn everything; his friend said that he had been in the same situation and had been taught on the job. This summer Chuck's buddy was going to be a chainman -- measuring distances, instead of holding the pole.

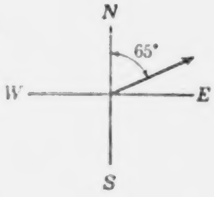
Chuck talked further and found that many people become surveyors after starting as rod or chainmen. Others go to a trade school for one-and-one-half years and get training in something like highway engineering. Many surveyors go to work for a government agency at a salary of either $7,300 or $8,100 a year with increases determined by experience and examination results. Private firms negotiate wages depending on need, experience, schooling, and other factors, but are usually close to those paid by the government.

SURVEYOR

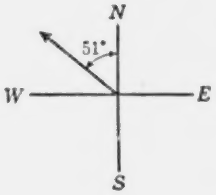
Chuck found that when a surveyor makes a "plot" drawing of a piece of ground he has surveyed, he has to make a great many calculations. He must take bearings (measurements in degrees) from various extablished points to lay out the plot. A north-south line is used as the standard and all readings are taken from it.

Study the following four figures.

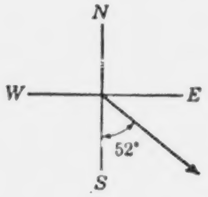
A



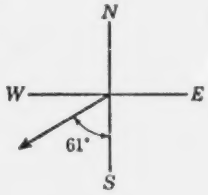
B



C



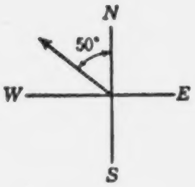
D



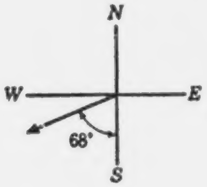
In figure A, the bearing of the line indicated by the arrow is 65° east of north, figure B, 51° west of north; figure C 52° east of south; and figure D, 61° west of south. For convenience, these bearings may be written fig. A, N 65°E; fig. B, N 51°W; fig. C, S 52°E; fig. D, S 61°W.

Read the bearings of the lines marked by arrows in E through H.

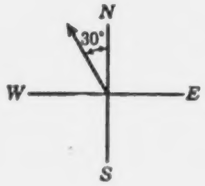
E



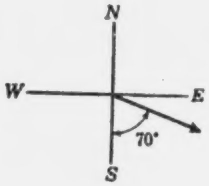
F



G



H



1. Fig. E ____________

2. Fig. F ____________

3. Fig. G ________________

4. Fig. H ________________

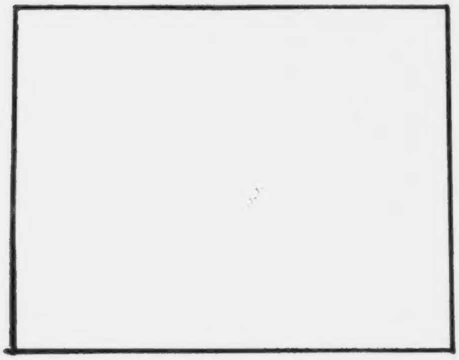
SURVEYOR

Surveyors must be able to read blueprint drawings and descriptions of property lines. A blueprint is a scaled drawing showing the length and direction of borders of property or the direction, widths, angles, etc. of roads. The scale used on the blueprint that is the unit of measure to be used on the blueprint for each actual unit of measure of the project, is determined by the size of the project and the desired size of the paper the blueprint is on. The amount of detail needed is also a factor in the size of the scale.

Example: A rectangular plot 16' by 24' could be scaled down to 16" by 24" on the drawing. The scale would be 1" = 1'. If the scale was 2" = 1', the drawing would be 32" by 48". If another plot of land was 75' by 30' and the scale for it is 1' = 5', the scaled drawing would be 5. a ______________________________

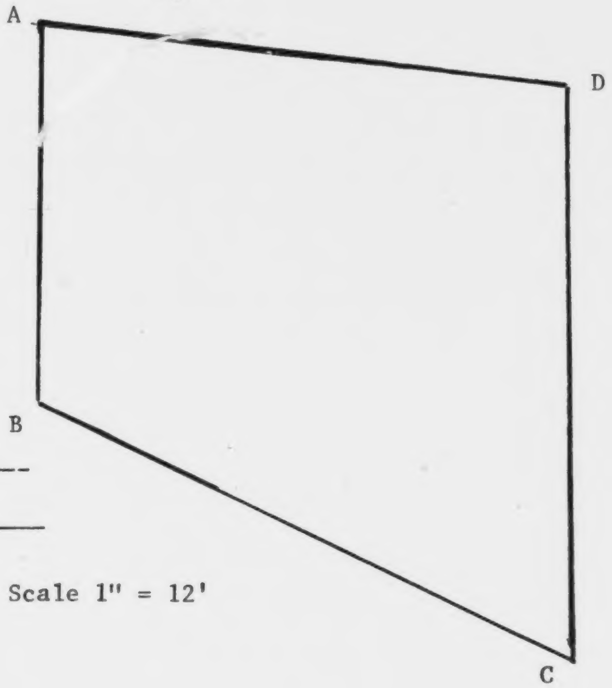
Find the actual length of each side of each of the following figures. Use a ruler to find the lengths of each line segment, then determine the actual length by using the given scale

fig. A



Scale 1" = 10'

fig. B



Scale 1" = 12'

b. Actual size of figure A ∠ = _____ W = _____

c. Actual size of figure B AB=________ BC=______

CD=________ DA=___________

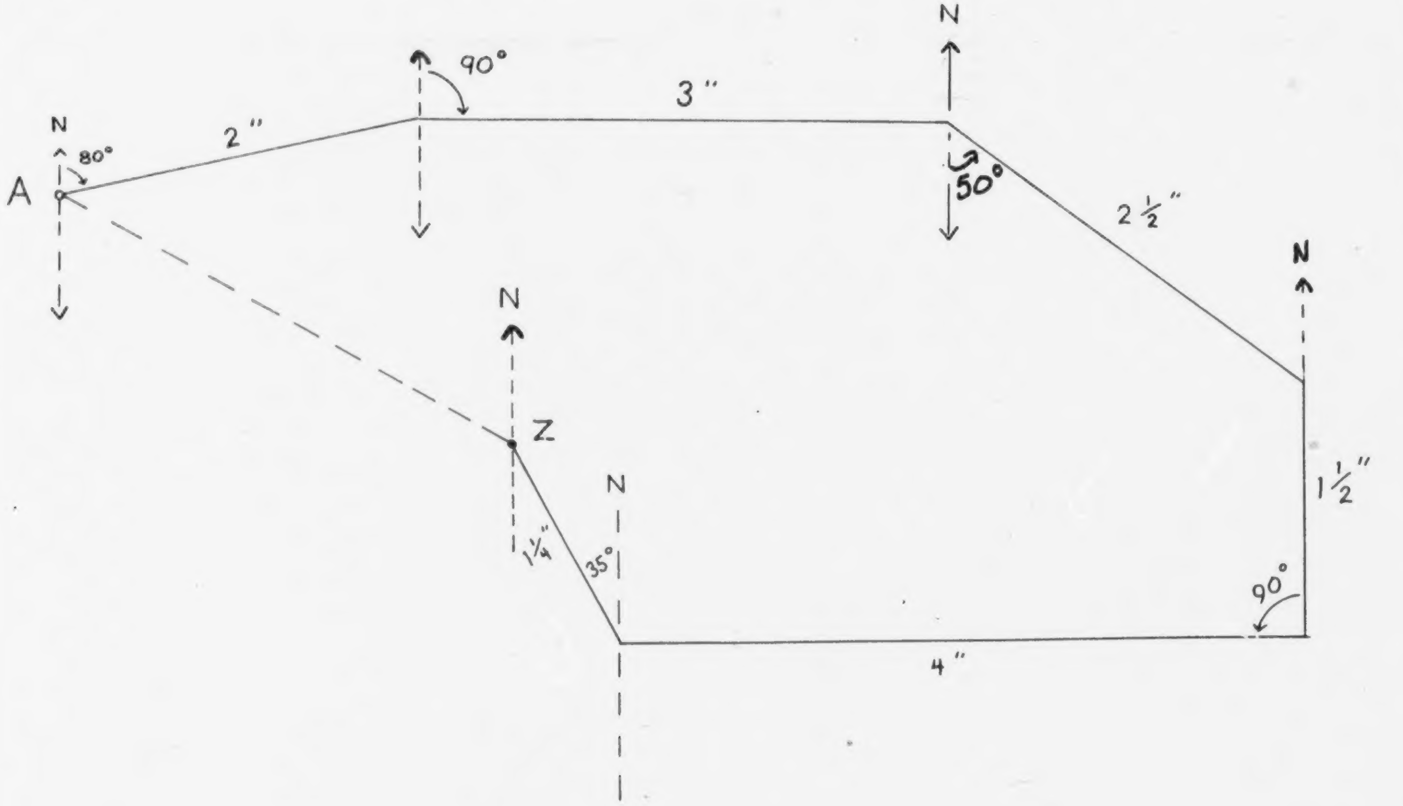
6. Combining a knowledge of bearings with that of scales, draw a diagram of the following property. The description begins from point A (northwest corner of property). (Use a protractor for measuring angles, a ruler for units. The scale is 1" = 50'.) From point A go N 45° E 100'; then S 90° E 150'; then S 45° E 125'; the S 0° 100'; then N 85° W 300'.

   a. From the last established point, find the distance to point A (approximate if not exact).

      ____

e. Also, using a protractor, find the bearning from the last established point to point A.________________________

Below is a drawing of a piece of property. Give a description of the property, as in the last problem, by giving directions and distance from each point starting at A. Scale: 1" = 50'



SURVEYOR

7. Decription of above property:____

____

____

8. What is the distance (approximated using the scale and your ruler) from point Z to A?____

9. Find the bearing reading from Z to point A.

____

Chuck felt the practical application of geometry in surveying was much more interesting than studying geometry from a book. The on-the-job aspect seemed to make geometry more useful.

II-21 Outdoor Advertising

OUTDOOR ADVERTISING

Bill came out of his geometry class shaking his head and muttering to himself. He had just learned that his semester grade would be dependent on a project on the uses of geometry in industry. Only last week Ed Zilensky, the class agitator, had tried to put Mr. Meier on the spot by asking him what good was anything they were studying. Mr. Meier had talked for some minutes on practical applications of geometry, but Bill didn't take notes because most of what he said wasn't too important -- so he thought. Now he wished that he had taken notes, for they would have come in handy. Mr. Meier had earlier told the class to expect a project of some sort, but the students were keeping quiet about it in hopes that he had forgotten. It must have been Ed's remarks that brought up the whole matter again.

Practical applications of geometry. Like what? Bill thought more about it as he drove home. He'd like something different and interesting, too! Casually he began humming "It's the real thing -----." "That 's it!" All around him he found billboards advertising all sorts of products. "Why not research the uses of geometry in outdoor advertising?"

Bill's first step was to contact one of the billboard companies in town. He was able to tour the plant and learn first-hand how billboards are produced.

In addition, he learned that advertising firms employ all kinds of different workers from executives to artists and layout specialists, salesmen, copywriters, and other administrative people.

Bill found that advertising firms have many different people of very different backgrounds earning extremely different salaries. They all, however, seemed to have artistic and language ability and interest in addition to an intense interest in working with people. These seemed to be the things that made for successful advertising people.

Managers, account executives, copywriters, media directors, production managers, and research directors were some of the people that Bill saw at work. They all make up a part of the 140,000 men and women working in the area of advertising. This number will not increase significantly during the next few years and competition for jobs will be keen through the 1970's. The highly qualified person, however, will always be in a good position to find employment in this field. "Highly qualified" means being experienced, creative, imaginative, and able to get along well with co-workers. This kind of person, Bill decided, would always be in a good position to find work.

As many as 120,000,000 people may view a poster in one day, depending on its location. Exposure is obvious and exposure is extremely important if one is going to sell a product. Secondly, communications is the name of the game. People need to be convinced that this product is really worth buying, and this communication must take place rapidly, since they're moving and will shortly move out of sight range.

There were basic principles to be followed and, though these were not strictly geometric, Bill thought he should list them:

7 Principles of Outdoor Design

At each step of development, from start to finish, check your design against these seven basic principles:

1. PRODUCT IDENTIFICATION
Does the advertiser's name or product register quickly?

2. SHORT COPY
Is the basic idea expressed quickly, and with impact?

3. SHORT WORDS
Can the reader grasp the idea at a glance?

4. LEGIBLE TYPE
Can the reader *read* the copy at a distance, while moving?

5. LARGE ILLUSTRATIONS
Are the pictures big as all Outdoor?

6. BOLD COLORS
Are colors clearly defined? Do they have impact?

7. SIMPLE BACKGROUND
Does the background interfere with the basic idea . . . or help it?

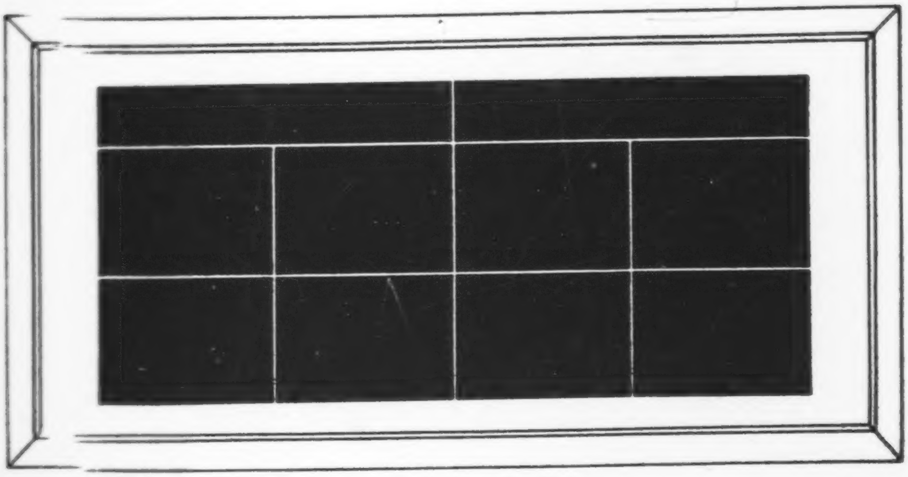
So far, Bill had been able to gather much interesting information but little which could be related to geometry. Outdoor advertising was definitely a form of art and governed by the rules of art more than by other more rigid guidelines. Color and contrast played very important roles in getting the message across in addition to the copy used, and some of this would be directly related to geometry.

However, the names posters and bulletins kept cropping up and Bill soon learned that there was a difference between them, although various designs could be used for both with minor alterations. Posters are prepared in the proportion 1 x $2\frac{1}{4}$ while bulletins are usually in the proportion 1 x $3\frac{1}{2}$ measuring 14' high and 48' long. As you can see, this proportion isn't exactly 1 x $3\frac{1}{2}$ for the bulletins but rather 1 x 1._____________; for practical purposes, though, 1 x $3\frac{1}{2}$ is close enough.

Remembering that posters run in the proportion 1 x $2\frac{1}{4}$ determine the correct sizes of copies submitted to the advertiser for approval: these usually run 6" x 2. _____" or 8" x 3._______". Finished artwork may be prepared in 12" x 4. _____", 16" x 5._______", or 20" x 6._____", all of which would maintain the same proportion. In many instances, a design may appear on both posters and bulletins.

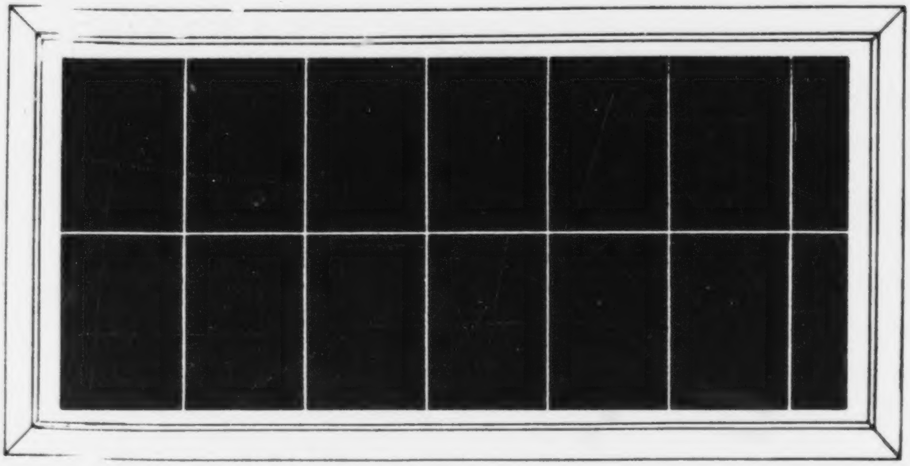
Posters come in three basic varieties:

I. 24-sheet poster



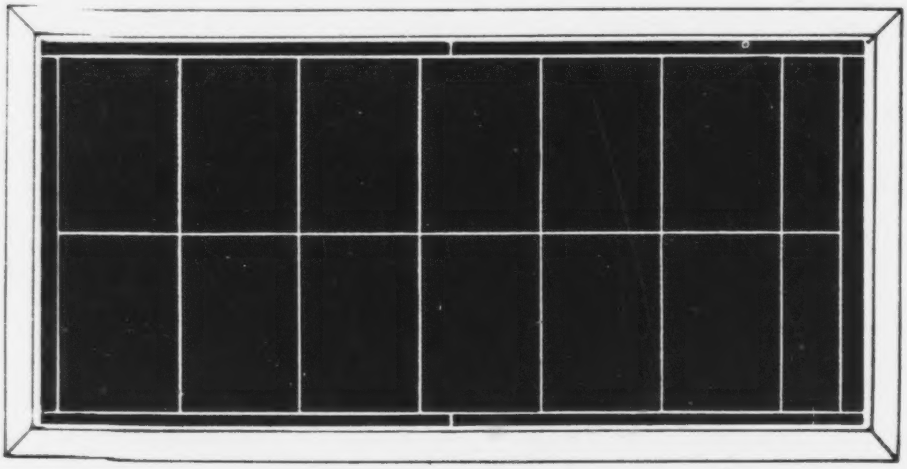
Originally, when presses were smaller, 24 sheets of paper were required for this. Larger sheets are now used, but the term remains. The area between the design and the frame is covered with white "blanking" paper.

II. 30-sheet poster



This gives about 25 percent more area for a design. Notice that the width of the "blanking" paper is quite reduced.

III. Bleed poster



This is the same as the 30-sheet poster except that the design is carried all the way to the frame. This is done by printing the "blanking" paper and offers 40 percent more design area than a 24-sheet.

OUTDOOR ADVERTISING

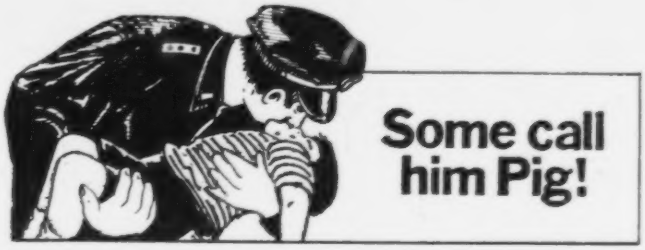
Outdoor posters are reproduced on paper by means of silk screen, letterpress, or lithography, depending on many factors, including the number of posters required. The three standard sizes have specifications given in the following table Use the information given to figure the total exposed area for the three types of posters and then check it against the standard poster ratio.

	24-SHEET	30-SHEET	BLEED
COPY AREA (length)	19′ 6″	21′ 7″	21′ 7″ Live Area
COPY AREA (height)	8′ 8″	9′ 7″	9′ 7″ Live Area
TOP & BOTTOM BLANKING	10½″ Top & Bottom (21″ total)	5″ Top & Bottom (10″ total)	5″ Top & Bottom (10″ total)
END BLANKING	19″ (38″ total)	6½″ (13″ total)	6½″ (13″ total)
FRAME WIDTH	11″	11″	11″

Kind	Exposed Area	Ratio
24-sheet	7.______ x 8.______	1 x 9.________
30-sheet	10.______ x 11.______	1 x 12.________
Bleed	13.______ x 14.______	1 x 15.________

Bulletins may also be in bleed form, depending upon the effect you wish to create. In addition, dramatic effects can be attained by extending certain elements of the design beyond the edge of the bulletin. The limits on any extension are on top: 5'6" extra; on each side and bottom: 2' extra each. From the dimensions given (14' by 48'), the total area of a bulletin is 16.____________. A further restriction on extensions is that the combined surface area of all space extensions must

not exceed 200 square feet. This would give a possible total area of 17.__________.
An example of an embellished bulletin would be:

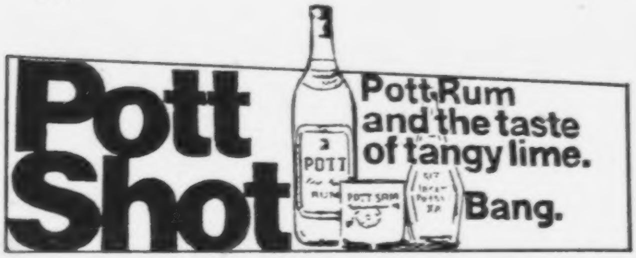


Try your skill at identifying the following as either posters or bulletins.

Find A) ratio, B) poster or bulletin, (check ratio) C) if poster: is it 24-, 30-sheet, or bleed

if bulletin: is it embellished?

18.



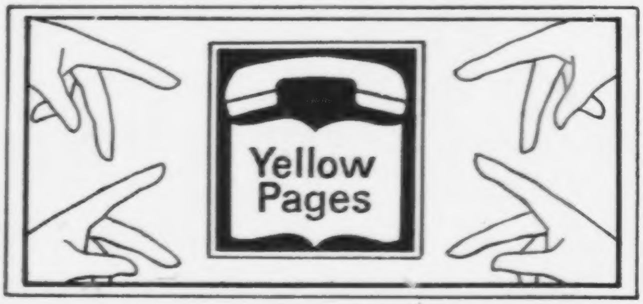
Advertiser:	Distim, Inc.
Agency:	Ingalls Assoc. inc., Ad.
Art Directors:	Jim Bennette/Karl Castleman
Copywriter:	Corso Danati

A. ________________

B. ________________

C. ________________

19.



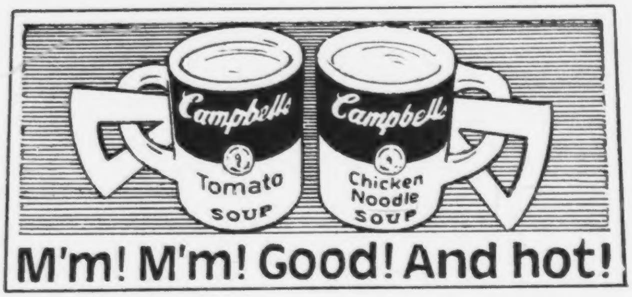
Advertiser:	Mich. Bell Telephone
Agency:	Ross Roy, Inc.
Art Director:	Robert Lawson
Copywriter:	Don Stoll

A. ________________

B. ________________

C. ________________

20.



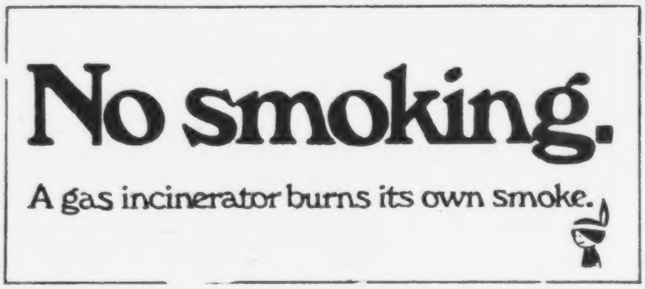
Advertiser: Campbell Soup Co.
Agency: BBDO
Art Direcor: Gene Bove
Copywriter: Mary Ellen Campbell

A. ____________

B. ____________

C. ____________

21.

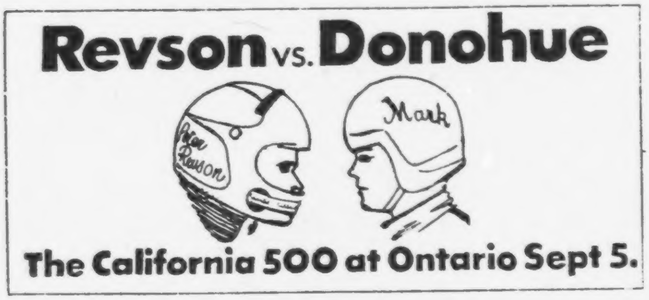

No smoking.
A gas incinerator burns its own smoke.

Advertiser: Minneapolis Gas Co.
Agency: Knox Reeves Advertising
Art Director: Tom Donovan
Copywriter: Mike Vukodinovich

A. ____________

B. ____________

C. ____________



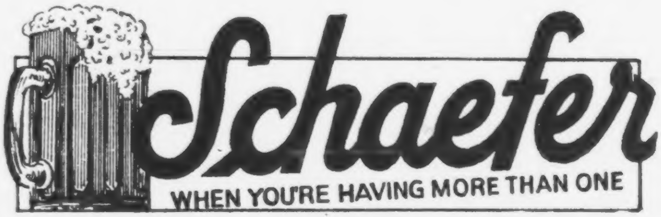
Advertiser: Ontario Motor Speedway
Agency: Chiat/Day Inc.
Art Director: Hy Yablonka
Copywriter: Jay Chiat

A. ____________

B. ____________

C. ____________

23.


Schaefer
WHEN YOU'RE HAVING MORE THAN ONE

Advertiser: Schaefer Beer
Agency: BBDO
Art Director: Bill Petti

A. ____________

B. ____________

C. ____________

Bill began to put his project together. He had discovered a lot about outdoor advertising that he didn't know before and it had been interesting researching his topic. Now all he had to do was prepare his project for submission to Mr. Meier. The fun was over. The work was about to begin.

II-22 Space

SPACE

Bruce looked out of the window as he wondered, "Who cares about all this stuff anyway? None of it's got anything to do with what I'm going to do."

It was obvious that Mr. Nelson thought otherwise about geometry, the way he was handling the unit. However, Bruces' thoughts were someplace else. He couldn't wait to get home and assemble the six-inch newtonian reflecting telescope that he had bought yesterday. He had done all sorts of odd jobs to raise the money and managed to con his dad into kicking in 20 percent of the cost. Actually, his dad had said that he couldn't raise the money and he had bet him the 20 percent that he could. They agreed to give his mom a different story because she frowned on gambling in any form. Anyway, Bruce had the telescope and was eager to get it in working order for tonight. He hoped to work directly in space exploration later if he could ever manage to get out of high school.

"Bruce, have you figured it out yet?" Mr. Nelson's voice shattered his train of thought. "Which project are you planning to work on?"

None of the available topics really excited him too much, so he asked Mr. Nelson if he could talk to him after class and decide then. Mr. Nelson was busy, but suggested a later hour. Since Bruce didn't have any classes then, they met in the classroom.

They had quite a discussion on the relevancy of geometry, especially as it related to Bruce's ambitions. The result of it was that Bruce's project for geometry was to show its uses in astronomy and space-related situations and to write up a report on a career in this area. "For once," thought Bruce, "I'm going to work on something I like."

It seemed reasonable to start by finding a few facts about an astronomer's job.

SPACE

Bruce wrote to the American Astronomical Society in New Jersey and got back some pamphlets. He discovered that astronomy is divided into many specialized branches. Bruce liked the astronomy which involves the "measurement of angular positions and movements of celestial bodies." Astrophysics, radio astronomy, and photometry were some of the other branches.

Bruce found that there aren't too many astronomers around and many of them have continued their schooling to very advanced degrees -- six to eight years beyond high school. He felt that if the courses he had to take were in the area of astronomy, even all that schooling wouldn't be bad. He did find that much of the schooling involved science and math related courses in addition to the elective classes.

The future for astronomers was interesting to Bruce. He found that since they are involved in the locations and orbits of planets, and they study the size and shape of the earth and its atmosphere and make calculations affecting navigation and measurement of time they will play a very important role. The whole area of space and programs involving satellites and exploration of space will also greatly affect this career area.

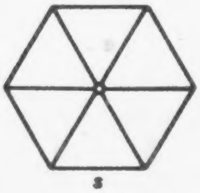
Bruce felt that with all the schooling required and the responsibilities, certainly he could expect to earn great money in that work! The average salary for those with a PHD degree, in 1970, was $15,000. This satisfied Bruce and he decided to start "zeroing in" on the geometry involved in space careers.

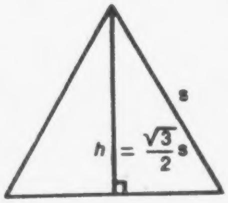
Bruce next found that problems range from the pure and oviously geometrical problems to some which seemed at first glance only vaguely related to geometry. In the first category would be problems concerning solar cells, area and load on the feet of a moon lanching craft, and the distance to the horizon from a given altitude above earth or the moon. Other problems based on geometry but also requiring other branches of mathematics for their solution would include transforming a rectangular map into an isosceles trapezoidal map, determining relationships of volumes and areas in spacecraft pressure and storage tanks, measuring the distance between earth and Mars. Following is a representative selection of what Bruce found.

1. Solar cells convert the energy of sunlight directly into electrical energy. For each square centimeter of solar cell in direct overhead sunlight, about 0.01 watt of electrical power is available. A solar cell in the shape of a regular hexagon is required to deliver 10.4 watts. Find the minimum length of a side.

Procedure

The total area required is 10.4 watts ÷ .01 watt per square centimeter, or a.______________square centimeters. The regular hexagon can be partitioned into six congruent triangles, each with an area of b.____________ square centimeters.





The area of an equilateral triangle with side s is: $A = \frac{s^2}{4}\sqrt{3}$

Solving this would give s = c.__________________

2. Solar cells are made in various shapes to use most of the lateral area of satellites. A certain circular solar cell with radius r will produce 5 watts. Two equivalent solar cells are made, one being a square with side s and the other an equilateral triangle with side p. Find r in terms of p and also in terms of s.

Procedure to find r in terms of s:

A (circle) = A (square)

$\pi r^2 = s^2$

Answer: r = a.____________________

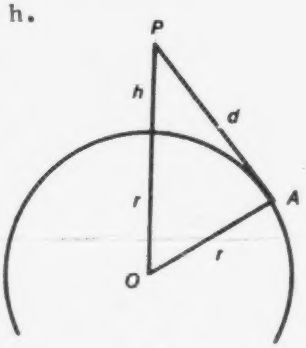
To find r in terms of p:

A (circle) = A (equilateral triangle).

Answer: r = b.____________________

3. A spacecraft is at P, at an altitude h above earth's surface, as pictured in the accompanying drawing. The distance to the horizon is d, and r is the radius of the earth.

   a. Derive an equation for d in terms of r and h.



Procedure

PA is tangent to the circle at A;

so $\angle$ PAO is a right angle

$r^2 + d^2 = (r+h)^2$ etc.

Answer: d = ____________________

   b. Find the distance to the horizon if h = 100 miles. Use 3,960 miles for the radius of earth.

Answer: d = ____________________

   c. It is apparent that for near-earth orbits, h will be small in comparison with r, so that discarding the h^2 term introduces only a small error. The formula then simplifies to $d = \sqrt{2rh}$. Find d with the simplified formula, and compute the percent of error that results when the h^2 term is dropped.

SPACE

Procedure

Use $d = \sqrt{2rh}$ and solve as in b above.

Answer, d = ____________________

answer of b   answer of c

To find percent of error: d. ______   e. ______ = 5.6

so that $\dfrac{5.6}{\text{answer of b}}$ = f. ______ percent.

4. Solve problem three with respect to the moon's horizon for a spacecraft 70 miles above the surface of the moon. Use 1,080 miles for the radius of the moon.

Procedure

Find true distance as in part b of problem three.

Answer: d = a.______________________________

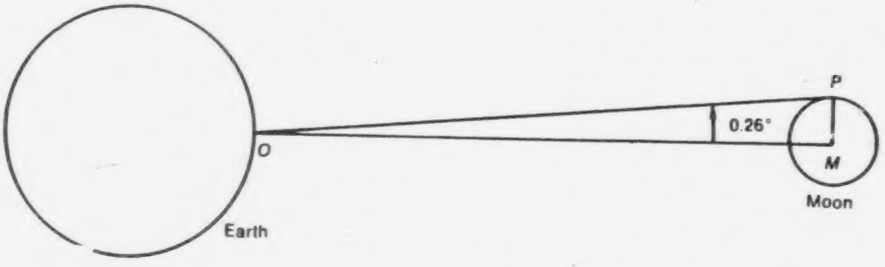
Find distance by simplified formula (as in part c)

Answer: d = b.______________________________

Find percent of error as in part c above.

Answer: c.______________________________

5. The average angle subtended by the moon for an observer on earth is 0.52° or 0.00907 radian. If the average distance from an observer on earth to the center of the moon is known to be 384,400 kilometers, find the diameter of the moon. Assume POM is a right triangle.



Procedure

Use the tangent function to find the radius of the moon, and then the diameter follows.

a.______________________________

Hint: $\text{Tan } 0.26^\circ = \frac{PM}{OM}$

Since $\text{Tan } 0.26^\circ = .00452$, PM may be determined.

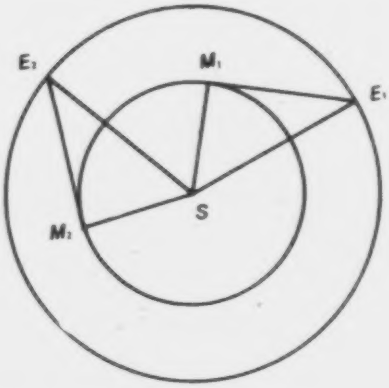
6. From earth, the planet Mercury appears to oscillate about the sun, appearing at elongation (its maximum angular distance from the sun as seen from earth) every 58 days. Earth and Mercury revolve in the same direction, counterclockwise as viewed from the north pole of the sun. Determine the period of revolution of the planet Mercury.

Procedure

An elongation occurs as SE_1M_1.

Fifty-eight days later, Mercury is at elongation on the other side of the sun and another 58 days later it is at the elongation SE_2M_2. During the 116 days, Mercury has traversed one revolution plus the arc M_1M_2. Earth has traversed in 116 days the angular distance $116 \times \frac{360^\circ}{365} = 114^\circ$ Now, the triangles SE_1M_1 and SE_2M_2 are congruent. Arc $E_1E_2 = 114^\circ$ and therefore arc $M_1M_2 = 114^\circ$. Hence, the period of Mercury is:

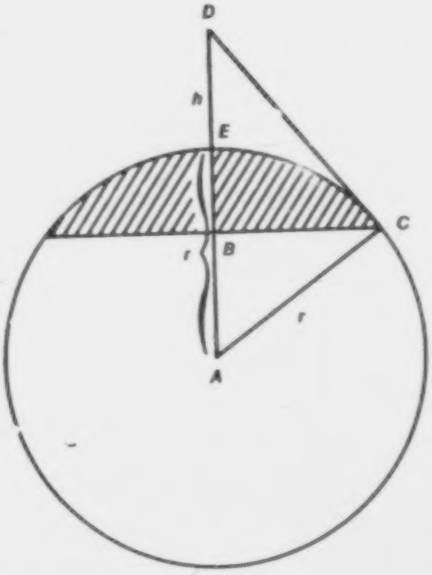
$360^\circ \times \frac{116 \text{ days}}{360^\circ + 114^\circ}$ = __________________days.



SPACE

7. Using the accompanying figure for these problems, take the radius of earth AE to be 3,960 miles. A formula has been derived for finding what fraction of the surface of a sphere of radius r can be seen from an altitude h above the surface of sphere. Let A_z be the area of the zone with altitude BE, A_e be the area of the earth, then

$$\frac{A_z}{A_e} = \frac{h}{2(r+h)}$$



Gemini 10, with astronauts Collins and Young aborad, flew in an orbit with perigee of 100 miles and apogee of 168 miles. What percent of earth's surface was visible from each of these two altitudes?

Procedure

Let h = 100; r = 3960, then $\frac{A_z}{A_e} = \frac{h}{2(r+h)} =$

Answer: a.________________

Let h = 168; r = 3960, then $\frac{A_z}{A_e} =$

Answer: b.________________

c. Find what altitude from earth the astronaut must be to see one-quarter of earth's surface at one time.

Procedure

Let $\frac{A_z}{A_e} = \frac{1}{4} = \frac{h}{2(3960 + h)}$ etc.

Answer: h = c.____________________

The first astronauts to travel that far from earth were Anders, Borman, and Lovell, on board Apollo 8, which orbited the moon on Christmas Day, 1968.

d. What percent of earth's surface can be "seen" from a synchronous satellite, whose altitude is 22,300 miles above earth?

Procedure

Let h = 22,300, $\frac{A_z}{A_e} =$

Answer ______________________

A synchronous satellite can relay messages to about e. ________ percent of the earth's surface. Thus three such satellites, evenly spaced around the earth over the equator, could form the basis of a communications network covering the entire earth